Parallel Nested Depth-First Searches for LTL Model Checking

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LIPN — Université Paris 13

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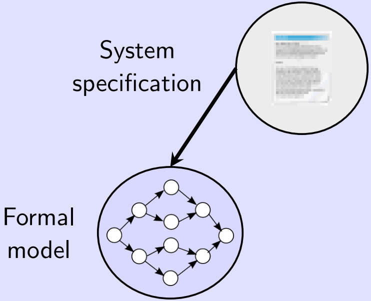
MeFoSyLoMa

Overview

The model checking approach

System specification

The model checking approach



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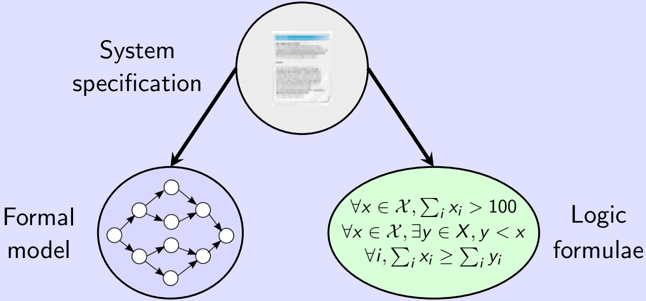

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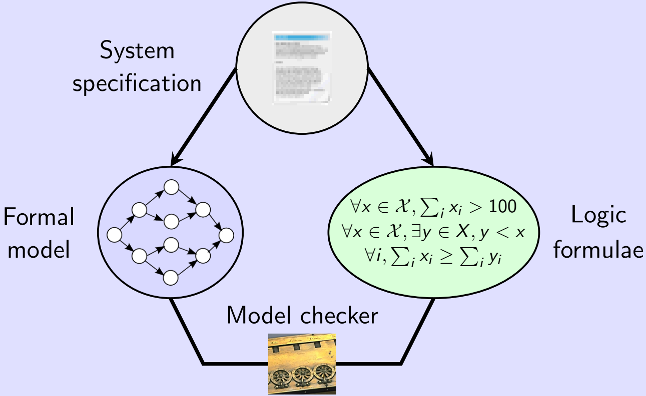

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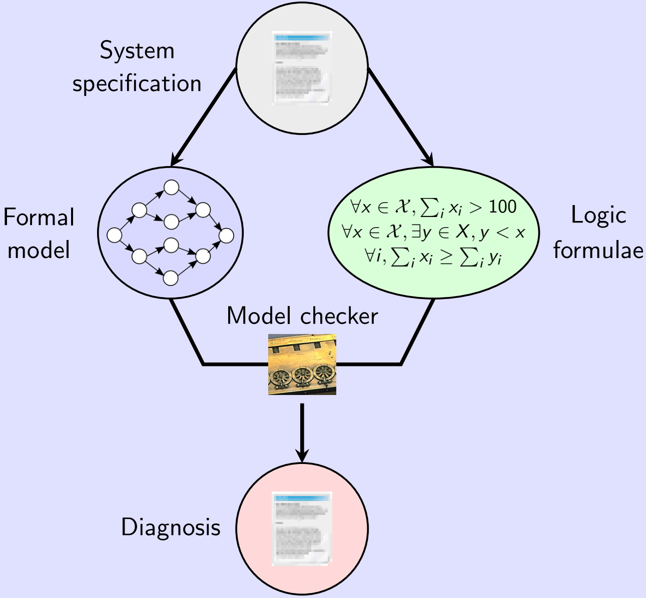

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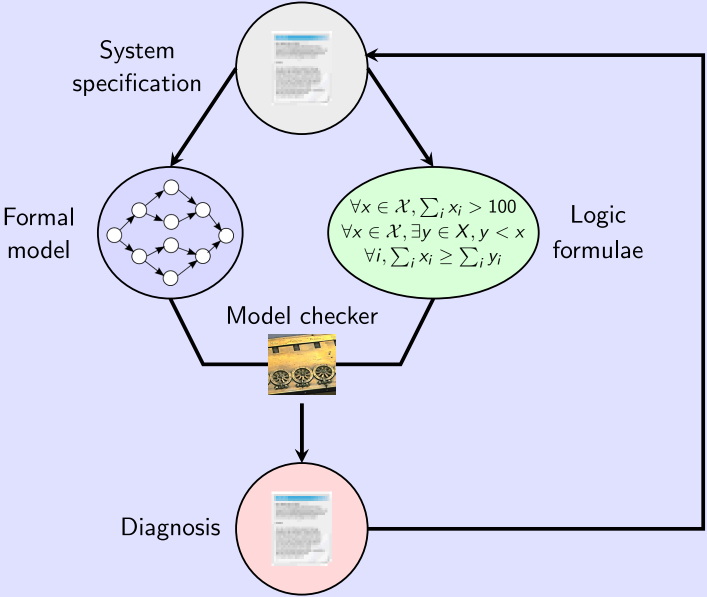

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- formal model $=$ directed graph $\mathcal{S}$ representing system's dynamic
- logic formula $=$ a property ϕ with temporal operators (e.g., until, next)

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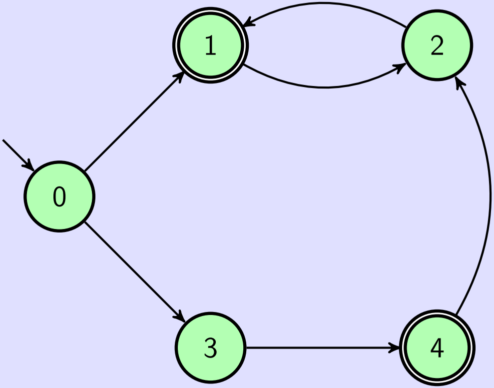
This talk

- an on-the-fly model checking algorithm (focussing on steps 2–3)
- for **multi-core** architectures with a **shared memory**
- adapted from classical nested depth-first search (used in SPIN)

Synchronized graph — An example



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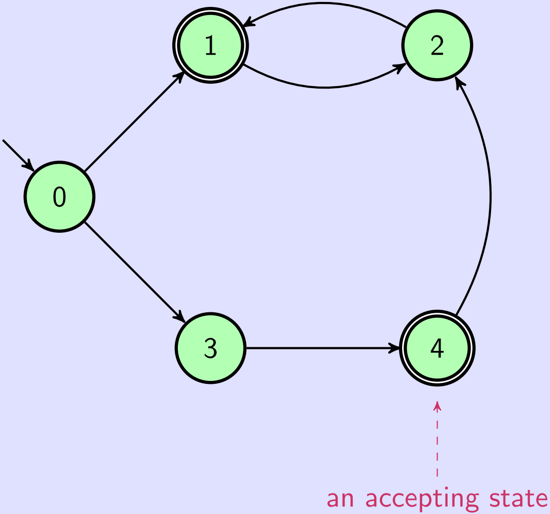

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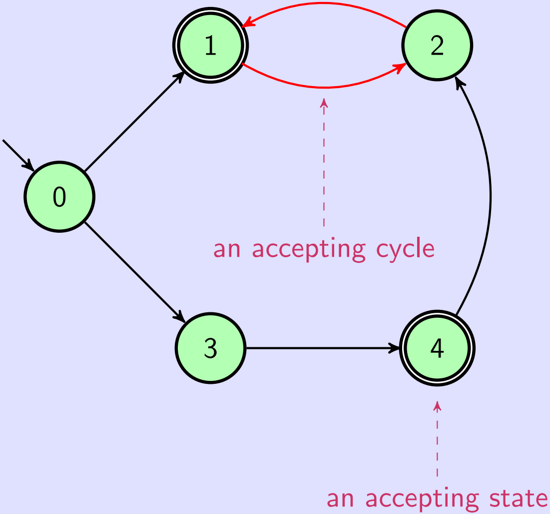

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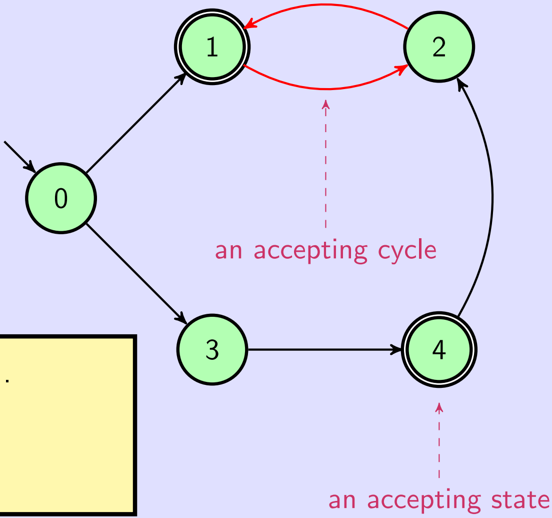

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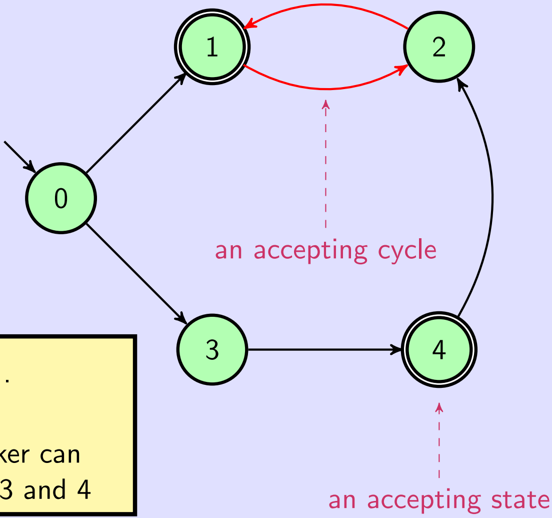

- execution 0, 1, 2, 1, 2, . . . invalidates the property
- an on-the-fly model checker can report it without visiting 3 and 4

Overview

Sequential algorithms for LTL model checking

Nested Depth-First search (ndfs)

Memory Efficient Algorithms for the Verification of Temporal Properties. CAV'1990. Courcoubetis, Vardi, Wolper and Yannakakis.

- Historical algorithm implemented by many model checkers, e.g., SPIN
- Principle: interleaving of two DFSs
  - a blue DFS that finds accepting states and launchs in DFS post-order
  - a red DFS that finds accepting cycles

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Strongly connected component based (scc-ltl)

On-the-Fly Verification of Linear Temporal Logic. FM'1999. Couvreur

Tarjan's Algorithm Makes On-the-Fly LTL Verification More Efficient. TACAS'2004. Geldenhuys and Valmari.

- Principle: adaptation of Tarjan's algorithm for finding SCC

Comparison of NDFS and SCC-LTL

- linear complexities for both
- ndfs uses less memory: 2 bits / state vs. 1–2 integers for scc-ltl
- but scc-ltl usually reports counter-examples faster.

⇒ both have their strengths

Experimental comparison in :

- **A Note on On-the-Fly Verification Algorithms**. TACAS'2005. Schwoon and Esparza.
- **On-the-Fly Emptiness Checks for Generalized Bchi Automata**. SPIN'2005. Couvreur, Duret-Lutz and Poitrenaud.
- **Comparison of Algorithms for Checking Emptiness on Büchi Automata**. MEMICS'2009. Gaiser and Schwoon.

Parallel Algorithms for LTL Model Checking

- most algorithms are designed for distributed memory architectures
- but these can be easily adapted to shared memory architectures
- usually rely on a BFS because DFS is hard to parallelize:

  Depth-First Search is Inherently Sequential. IPL'1985. Reif.
- characterized by different "on-the-flyness" levels:
  0 off-line: first we build the synchronised graph then we check
  1 early termination possible but not guaranteed in the presence of an accepting cycle
  2 on-the-fly: early termination guaranteed in the presence of an accepting cycle

Multi-Core Algorithms for LTL Model Checking

Algo.	Source	Time Comp.	Proc.	Acceleration	OTF
2-ndfs	TSE'07	$\mathcal{O}(n+m)$	1–2	average	2
map	FMCAD'04	$\mathcal{O}(a^2 \cdot m)$	1–N	excellent	1
owcty	SPIN'03	$\mathcal{O}(h \cdot m)$	1–N	excellent	0
owcty-otf	ICFEM'09	$\mathcal{O}(h \cdot (m+n))$	1–N	excelent	1
negc	FSTTCS'01	$\mathcal{O}(n \cdot m)$	1–N	excellent	0
bledge	ASE'03	$\mathcal{O}(m \cdot (n+m))$	1–N	excellent	0
bledge-otf	ENTCS'05	$\mathcal{O}(m \cdot (n+m))$	1–N	excellent	2
mc-ndfs	this talk	$\mathcal{O}(p \cdot (n+m))$	1–N	average-good	2

- n, m = states and edges in the graph
- a = accepting states
- h = height of the graph
- p = working processes
- acceleration obtained through experimentations
- OTF = on-the-flyness

Overview

Principle of NDFS

- based on two DFSs
- a **blue** DFS is used to find all accepting states
- when it backtracks from an accepting state a the **red** DFS is initiated
- the red DFS tries to find a way back to a
  a is called a seed state
- each state is explored at most twice (by the blue or red DFS)
- requires two bits per state to remember explored states

Pseudo-code of the algorithm

Main procedure

```
for each state s
   s.blue := false
   s.red  := false

dfsBlue (initial state)
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The blue DFS

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dfsBlue (s)
   s.blue := true
   for (s' in succ (s))
      if (not s'.blue)
         dfsBlue (s')
   if (s is accepting)
      seed := s
      dfsRed (s)
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      seed := s
      dfsRed (s)
```
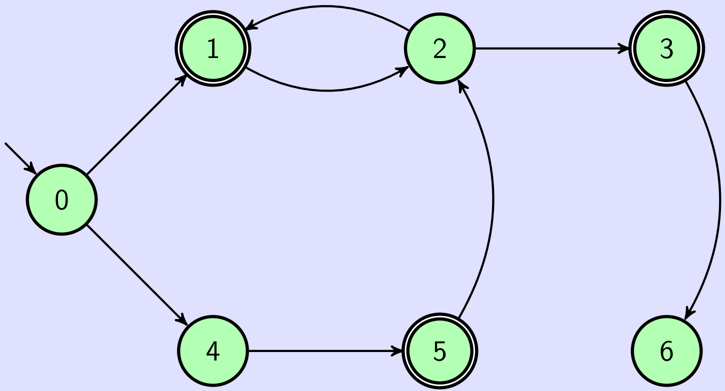

The red DFS

```
dfsRed (s)
   s.red := true
   for (s’ in succ (s))
      if (s’ = seed)
         print "Cycle Found"
      else if (not s’.red)
         dfsRed (s’)
```

NDFS — Example 1



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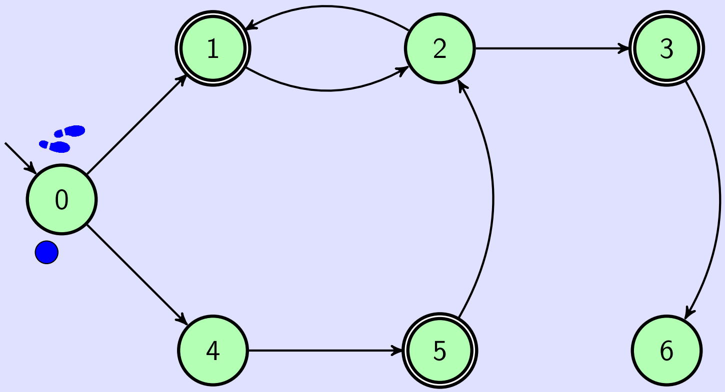

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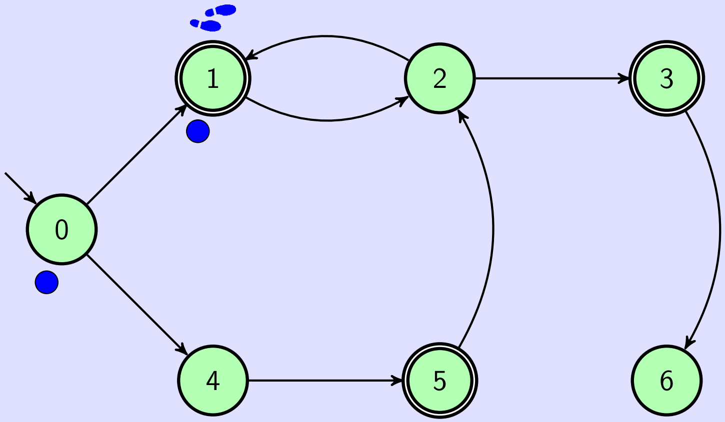

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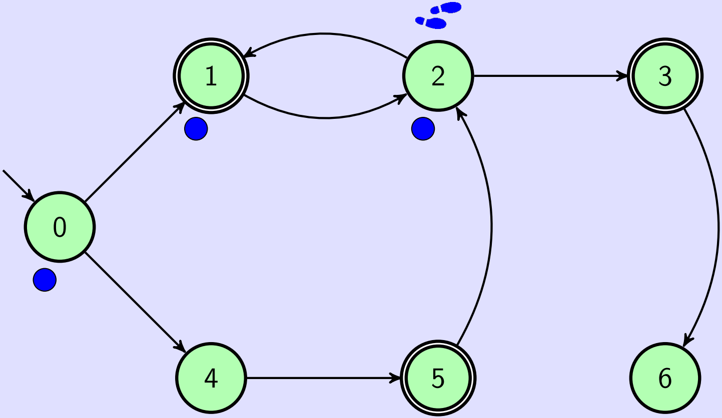

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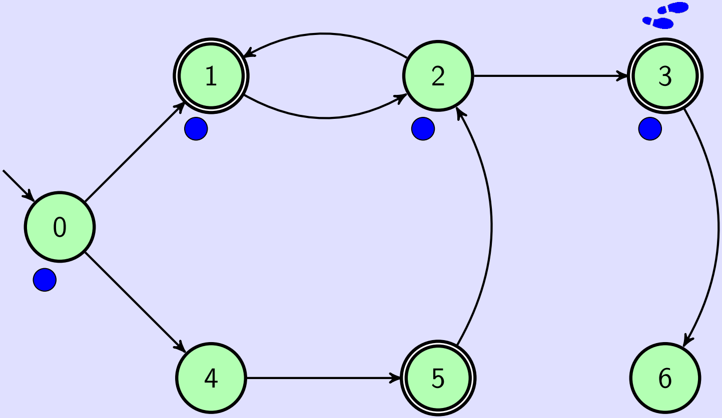

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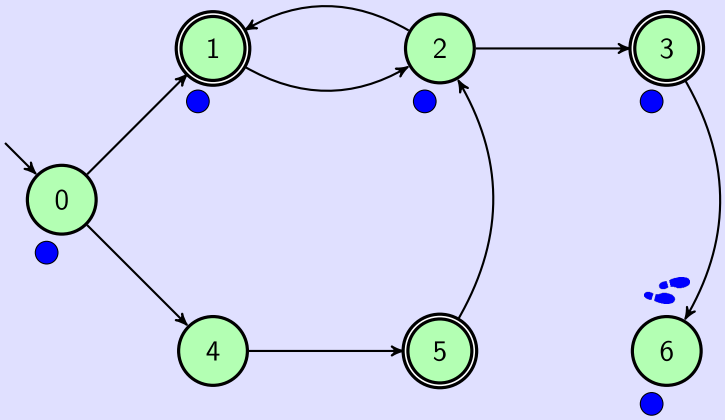

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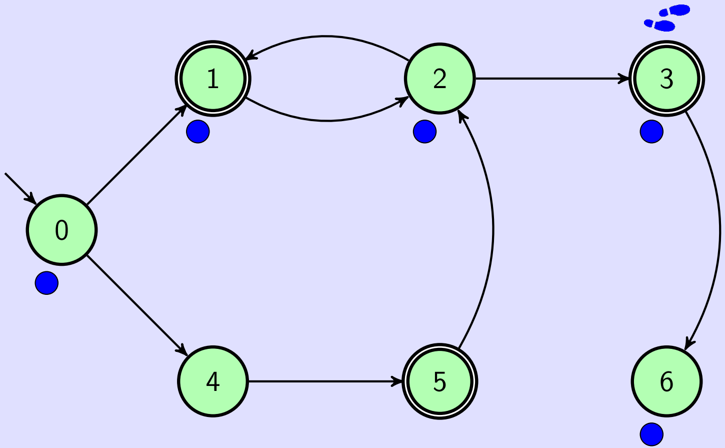

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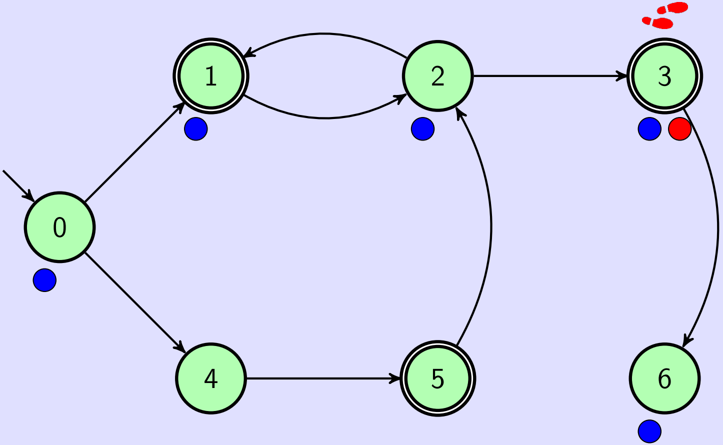

start a red DFS with 3 as seed

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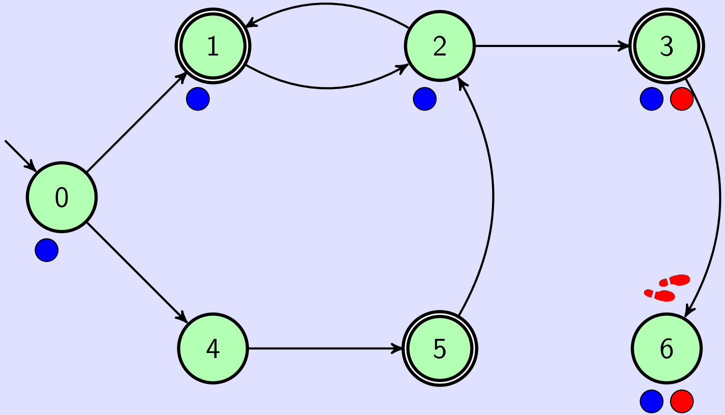

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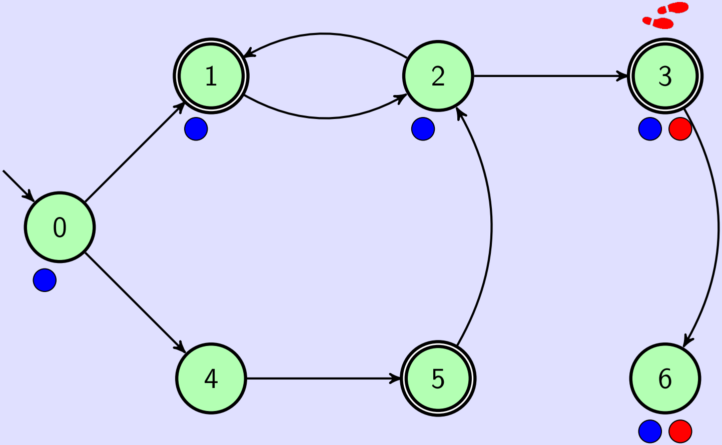

the red DFS terminates ⇒
no accepting cycle around 3

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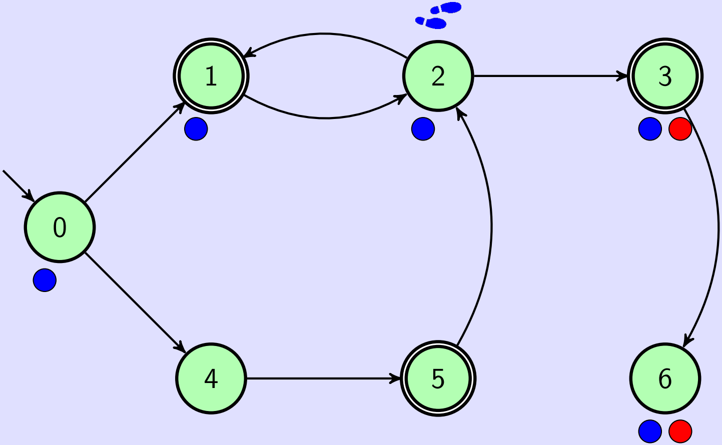

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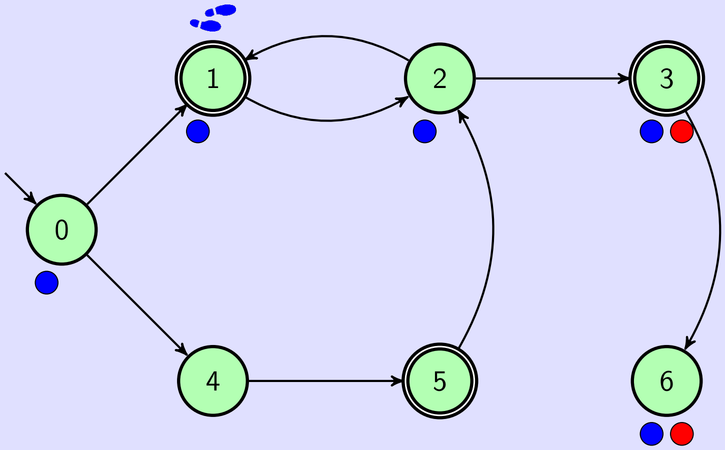

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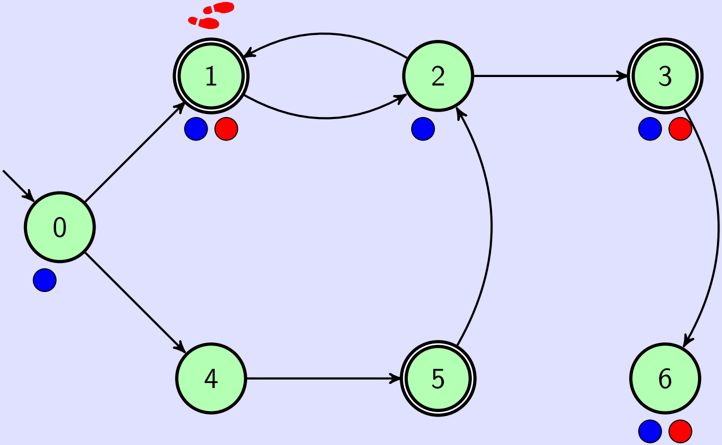

start a red DFS with 1 as seed

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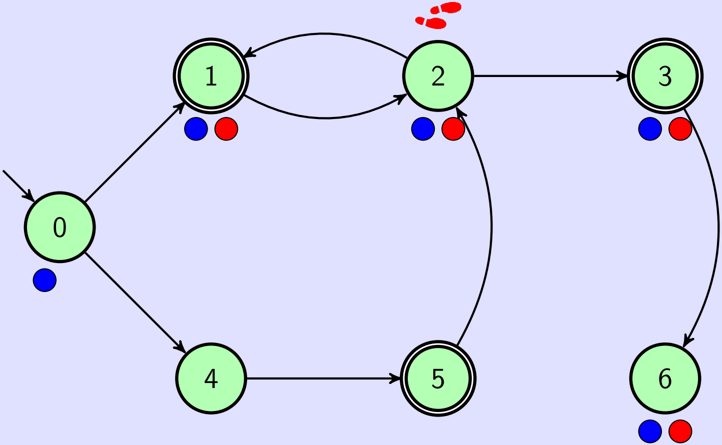

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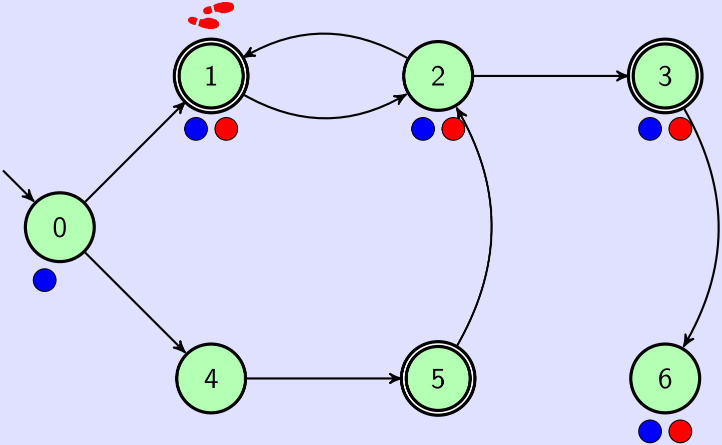

we reach the seed ⇒
accepting cycle found

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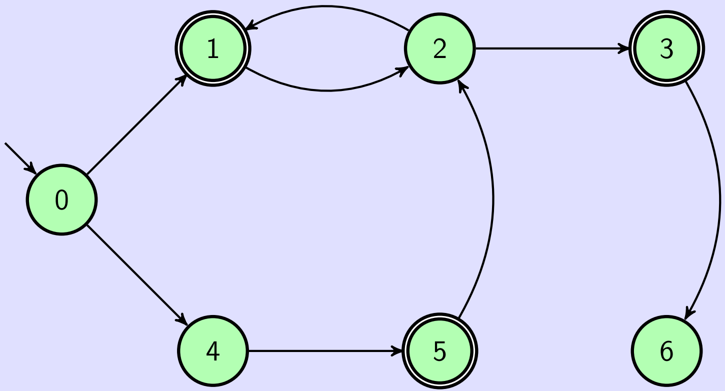

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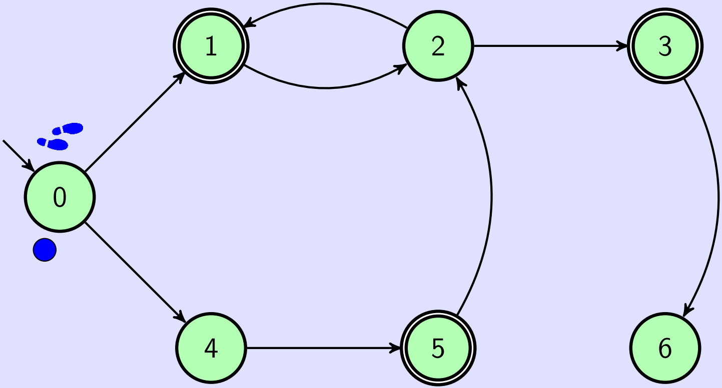

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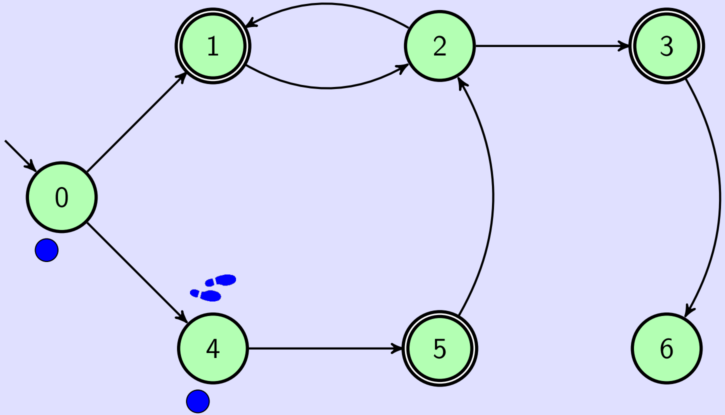

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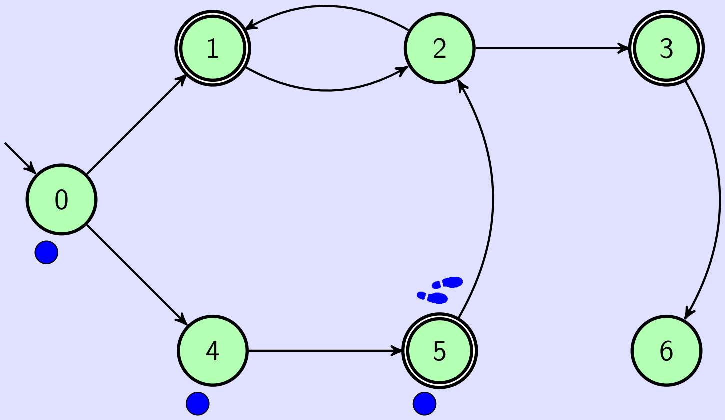

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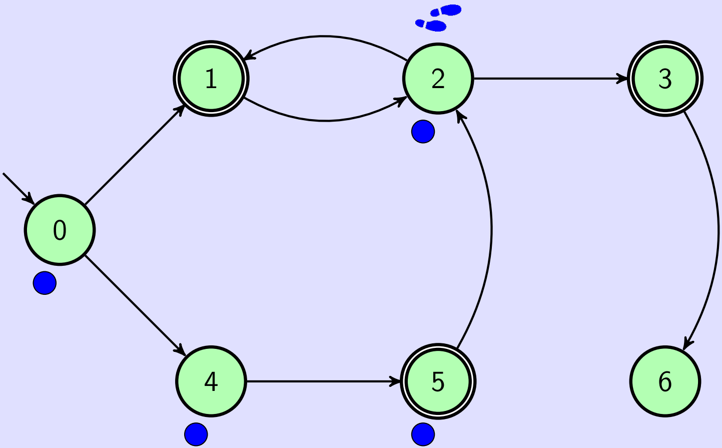

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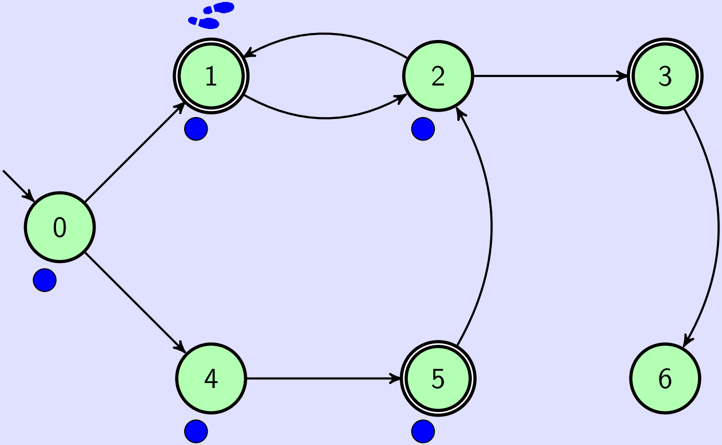

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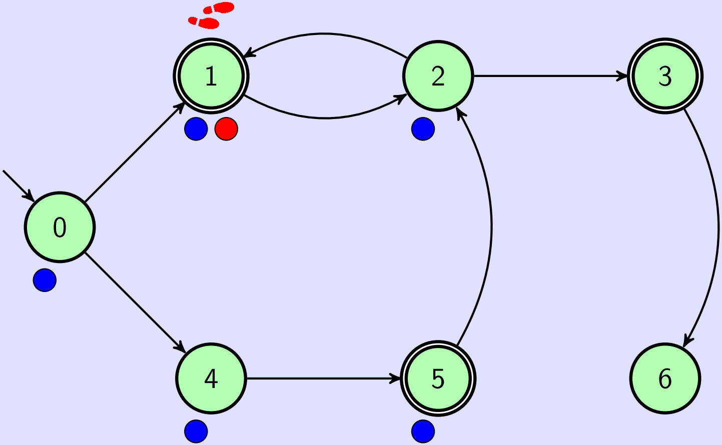

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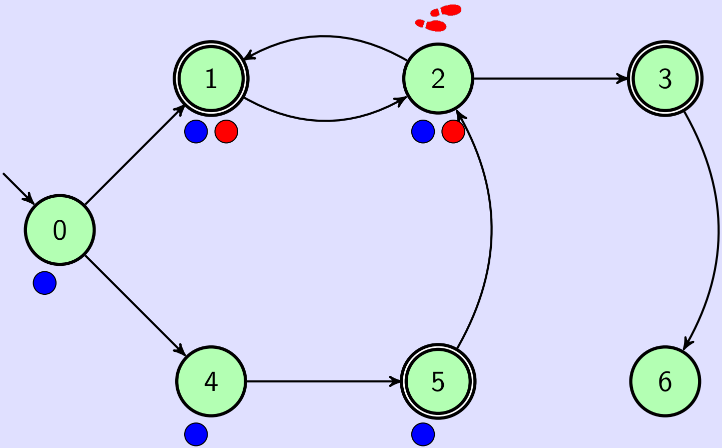

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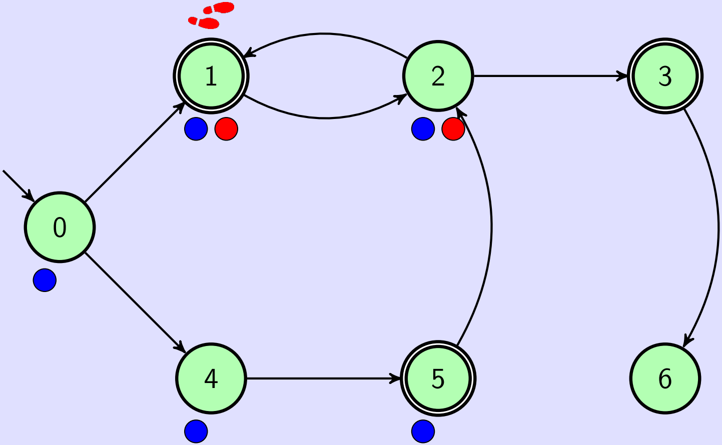

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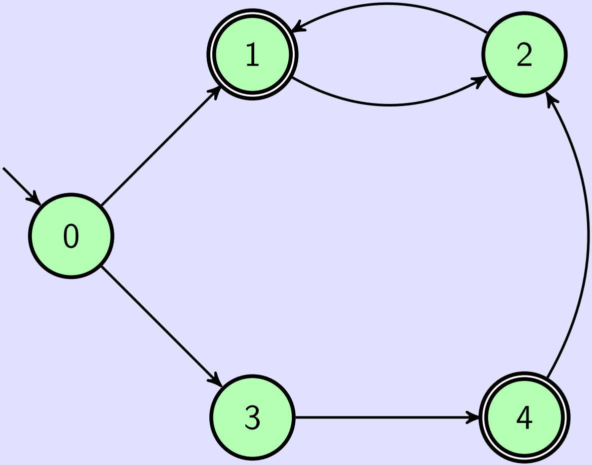
our starting point: ndfs

Why is it difficult to parallelize ndfs?

a naive multi-threaded version of ndfs:

- threads are launched concurrently
- each thread performs the ndfs algorithm
- threads share all blue and red bits of ndfs

Why is it difficult to parallelize ndfs? — An example



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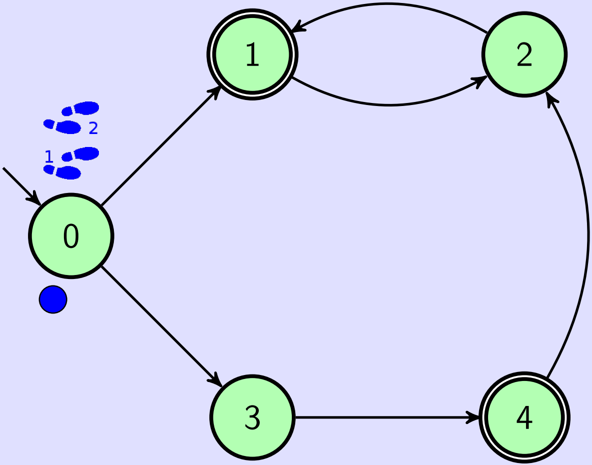

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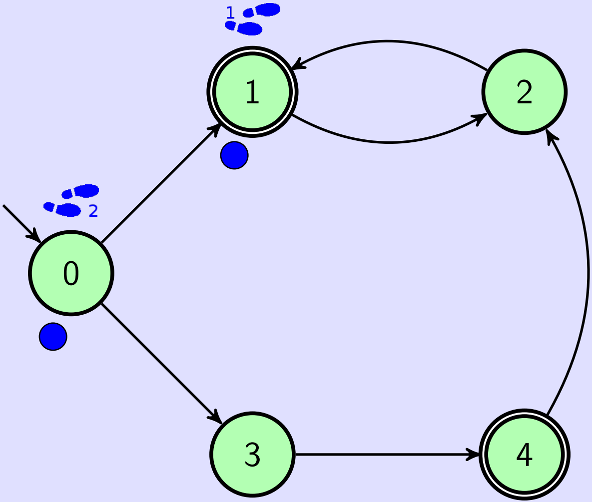

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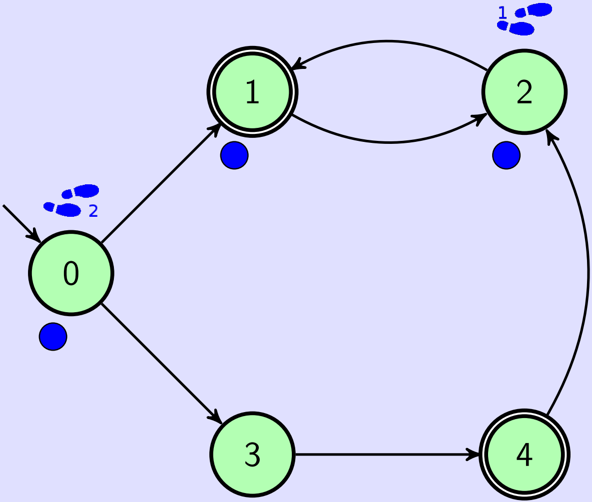

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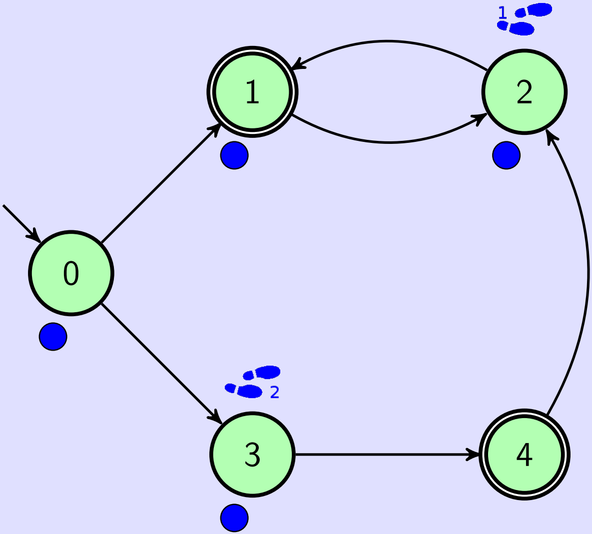

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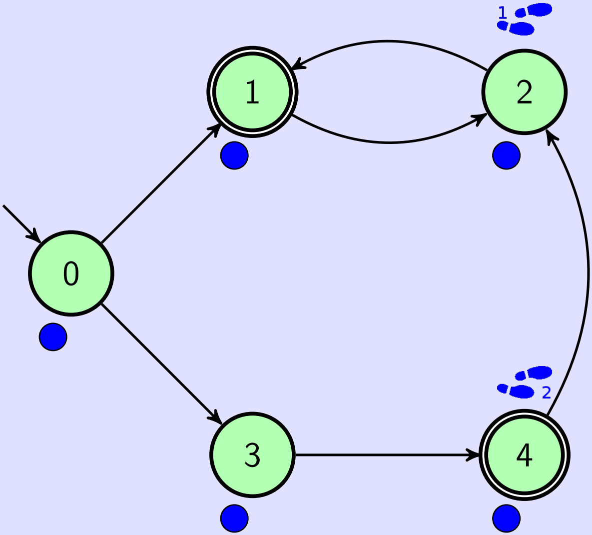

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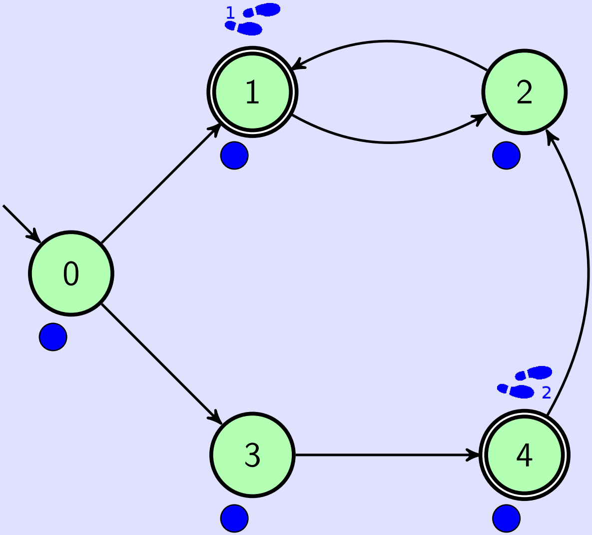
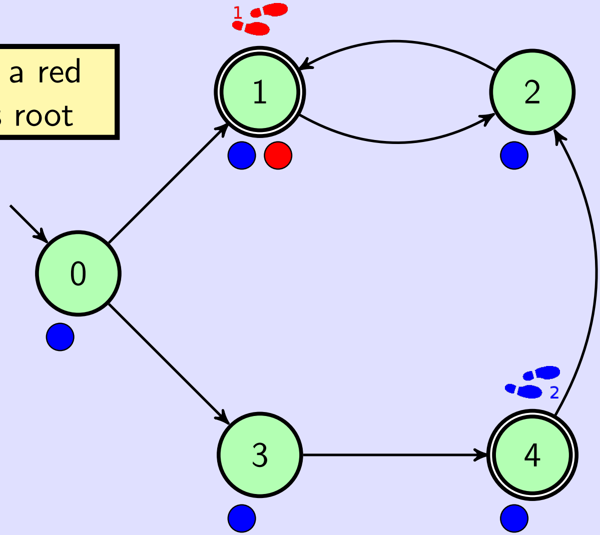


Why is it difficult to parallelize ndfs? — An example

thread 1 starts a red DFS with 1 as root



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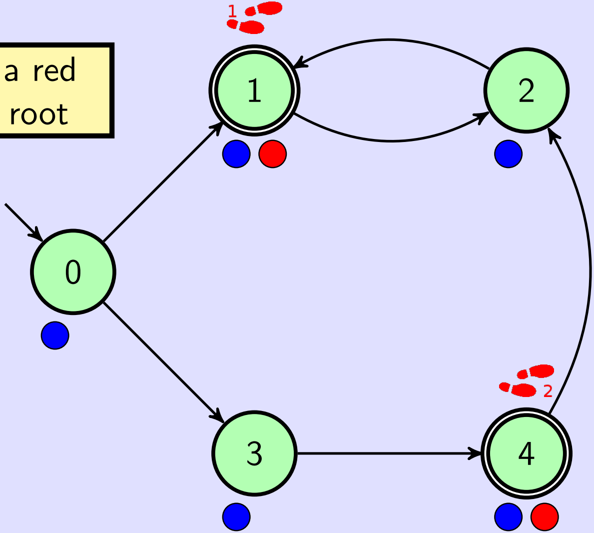
thread 2 starts a red DFS with 4 as root



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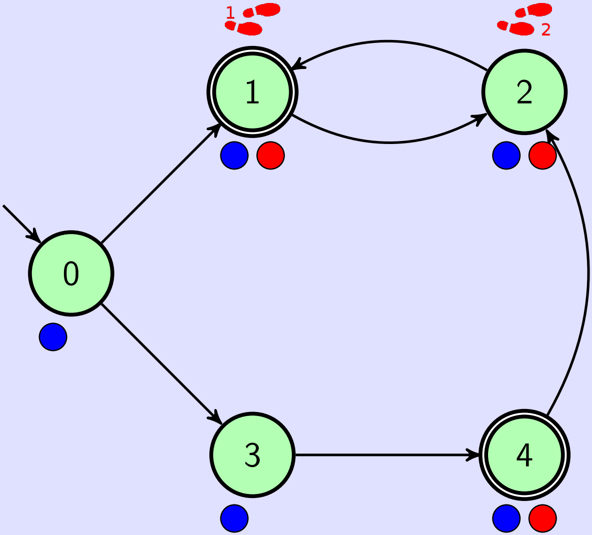

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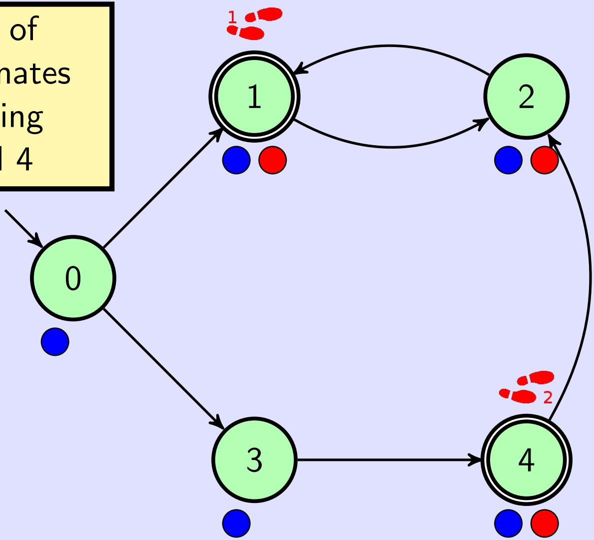
the red DFS of
thread 2 terminates
$\Rightarrow$ no accepting
cycle around 4



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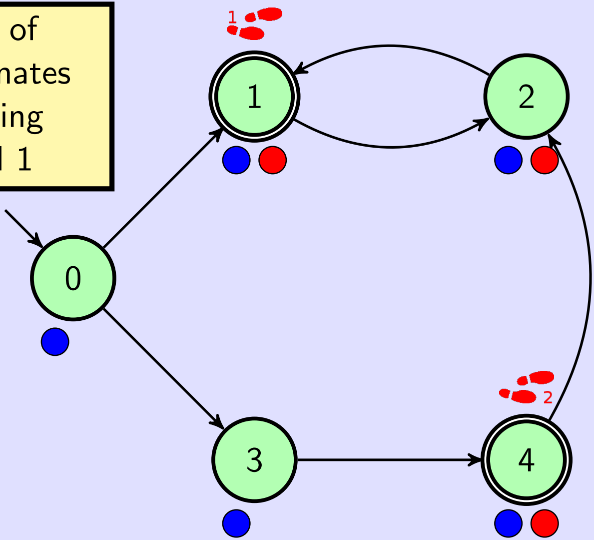
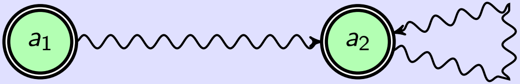
⇒ no accepting
cycle around 1



Why is it difficult to parallelize ndfs?

- because the invokation order of the red DFS is important!
- basically if we have two accepting states a_1 and a_2 with
  - $a_1 \rightsquigarrow a_2 \wedge \neg a_2 \rightsquigarrow a_1$
  - $a_1 \notin$ an accepting cycle and $a_2 \in$ an accepting cycle:



- then $dfsRed(a_1)$ must be invoked **after** $dfsRed(a_2)$
  - otherwise $dfsRed(a_1)$ will mark states around a_2 as red
  - and $dfsRed(a_2)$ will not discover the accepting cycle around a_2
- we call this situation a **conflict**
- why does ndfs work? because the red DFS is nested in the blue DFS

$\Rightarrow$ $dfsRed(a_2)$ will be invoked **before** $dfsRed(a_1)$

- but a naive multi-core ndfs does not preserve this order

Principle of MC-NDFS

- mc-ndfs = multi-core ndfs
- mc-ndfs spawns multiple threads that all execute (a modified) ndfs
- exploration based on 2 principles: **randomisation** and **synchronisation**
- randomisation: threads explore the graph in a random way so that they (hopefully) engage in different parts of the graph
- synchronisation: shared memory is used to avoid as much as possible redundant revisits by different threads

Resolution of conflicts

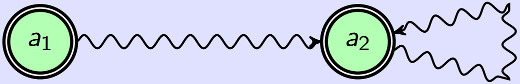
- mc-ndfs follows an **optimistic** approach to resolve conflicts:
  - we let threads explore the graph without taking care of conflicts
  - there is a way to detect when these conflicts occur
  - in that case, a thread relaunchs a red DFS by only using local data
  - why?
    - because shared attributes modified by other threads have corrupted the result of a red DFS launched
    - using only local data we simulate ndfs and are thus on the safe side
- thus we have two layers algorithm
  - a **multi-core** layer with inter-process synchronisation
  - an **emergency** (without synchronisation) layer triggered in case of conflict

✔ we avoid all synchronisations/waitings due to the prevention of conflicts

✘ in case of conflicts, states will be revisited multiple times

How do we detect and fix conflicts?

- a conflict occurs when a red DFS initiated on a_1 reaches a_2 that is
  - accepting
  - but not red ($\Rightarrow$ the red DFS on a_2 has necessarily not terminated)
- this situation possibly corresponds to a conflict:

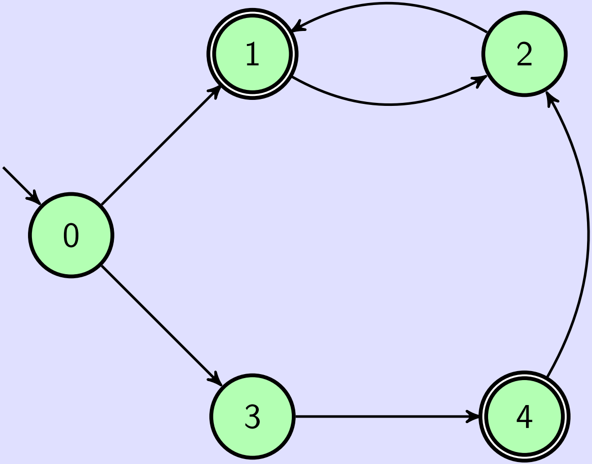


- what do we do then? we mark a_2 as **dangerous**

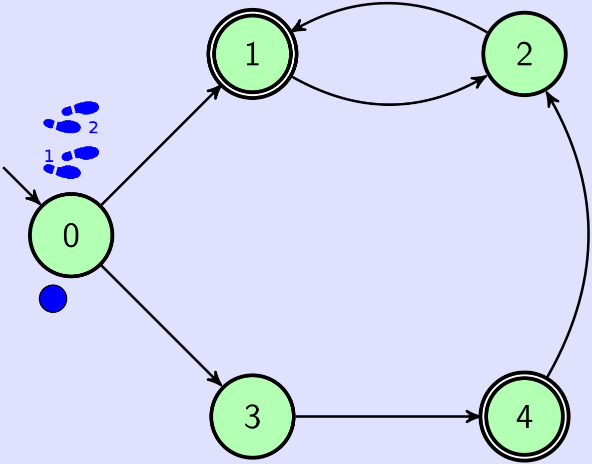
$\Rightarrow$ this means that the emergency level must be triggered for a_2

- the red DFS that will be initiated on a_2 does not report an accepting cycle $\Rightarrow$ relaunch a red DFS on a_2 and ignore global red flags

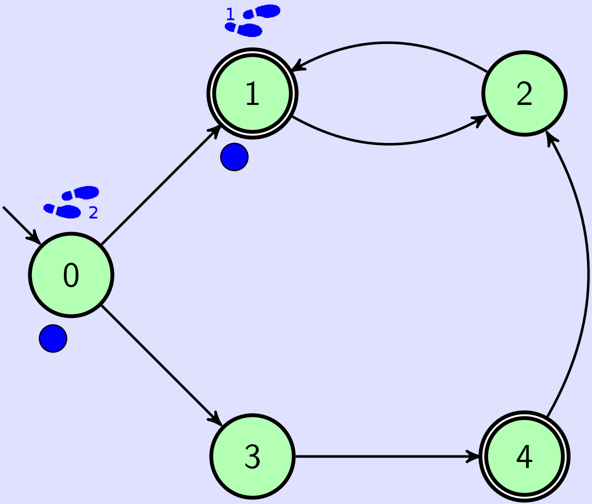
Example



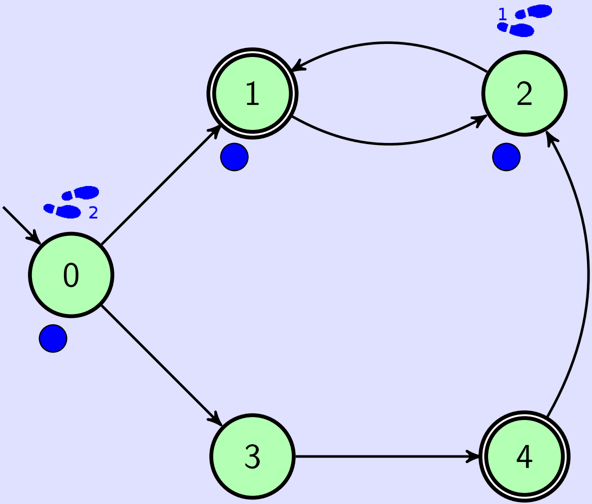
Example



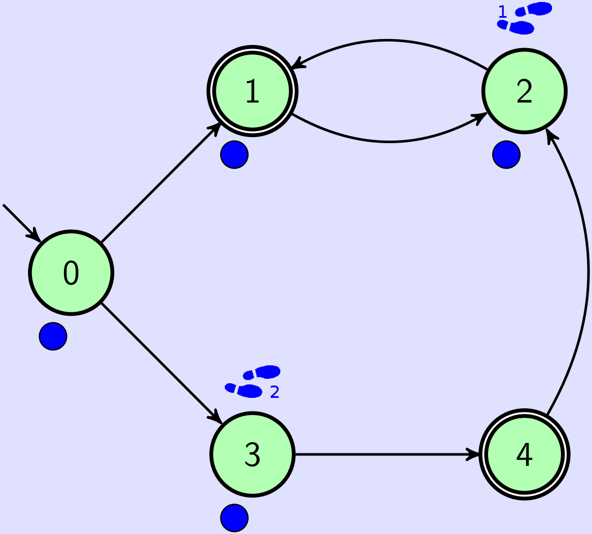
Example



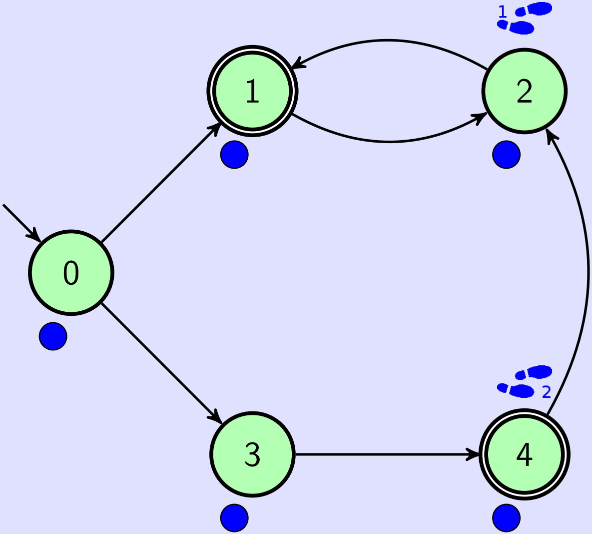
Example



Example

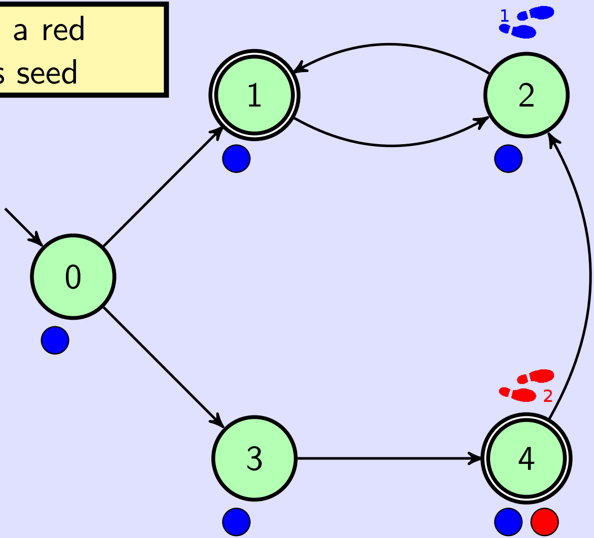


Example

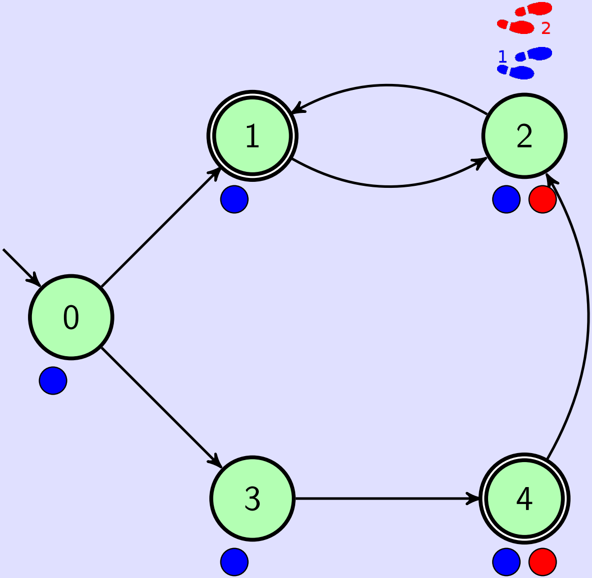


Example

thread 2 starts a red DFS with 4 as seed

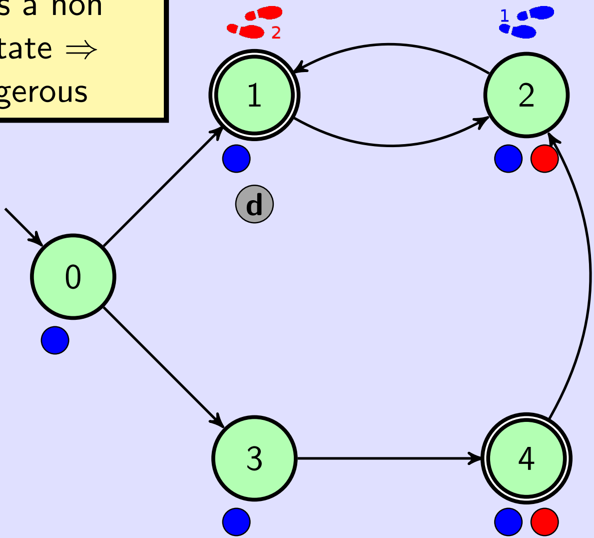


Example

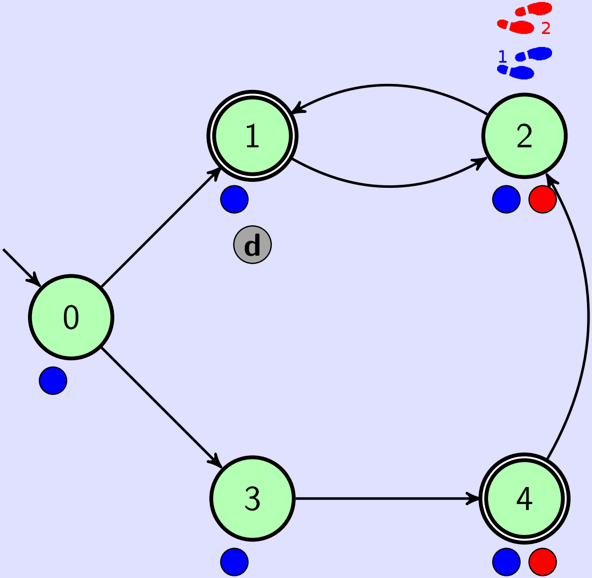


Example

thread 2 reaches a non red accepting state ⇒ mark 1 as dangerous

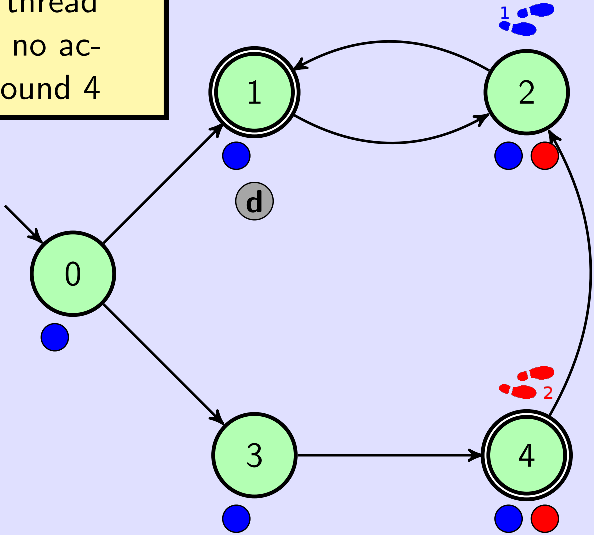


Example

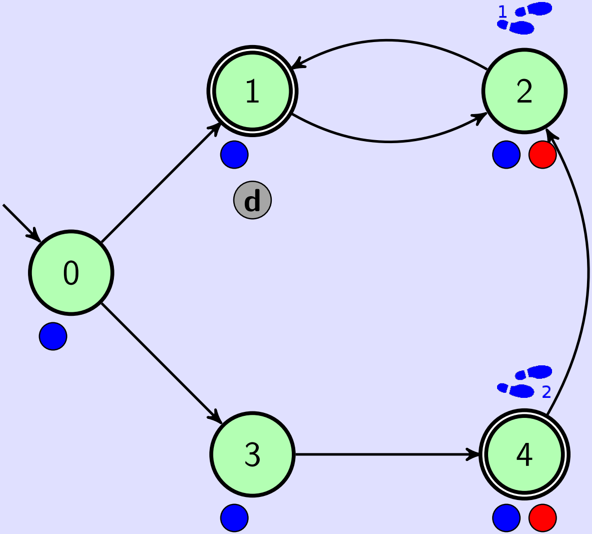


Example

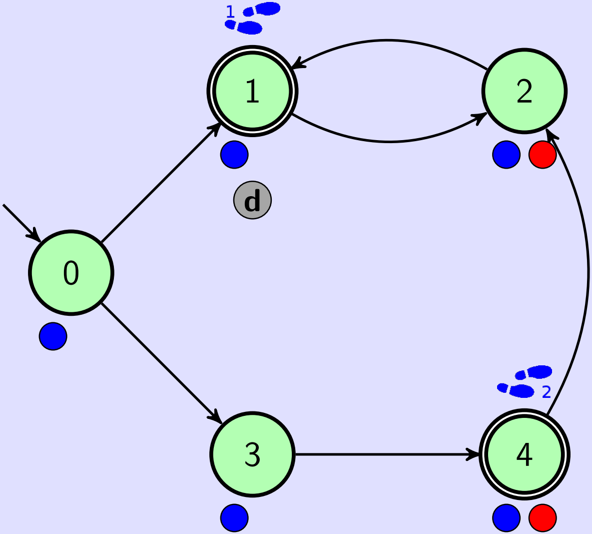
the red DFS of thread 2 terminates ⇒ no accepting cycle around 4



Example

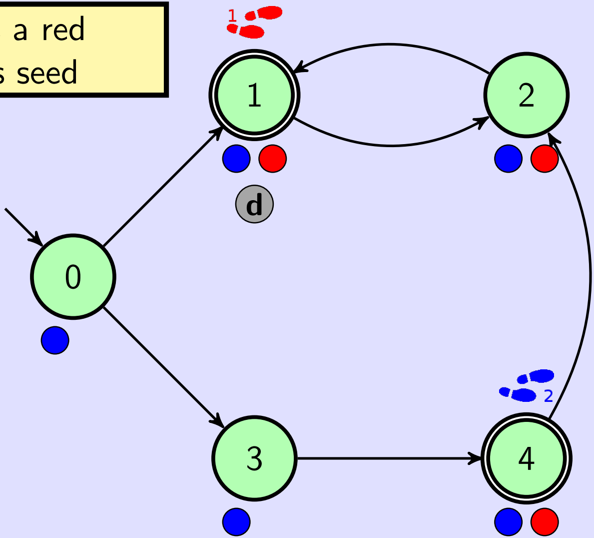


Example



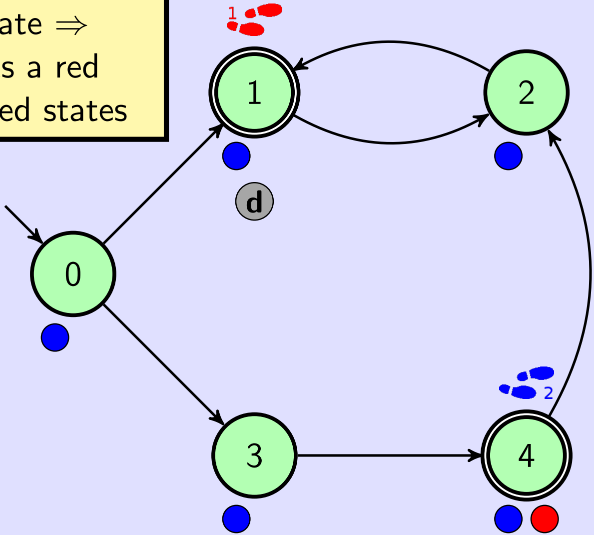
Example

thread 1 starts a red
DFS with 1 as seed

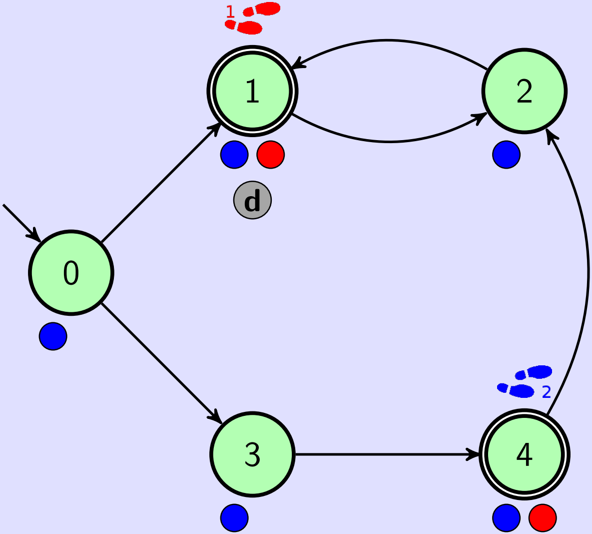


Example

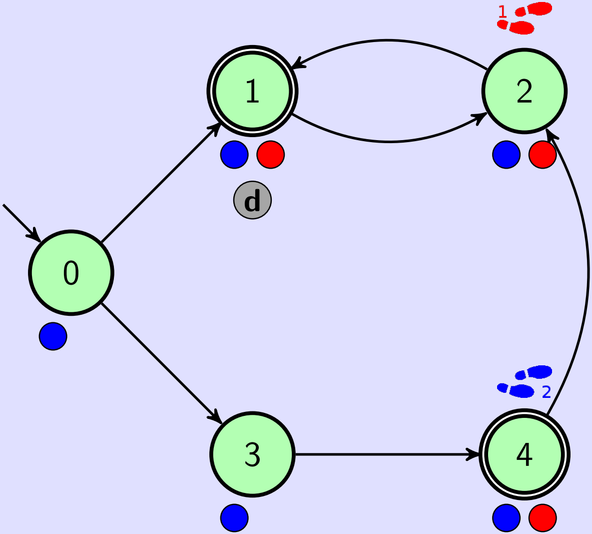
red DFS terminated on a dangerous state ⇒ thread 1 restarts a red DFS and ignore red states



Example

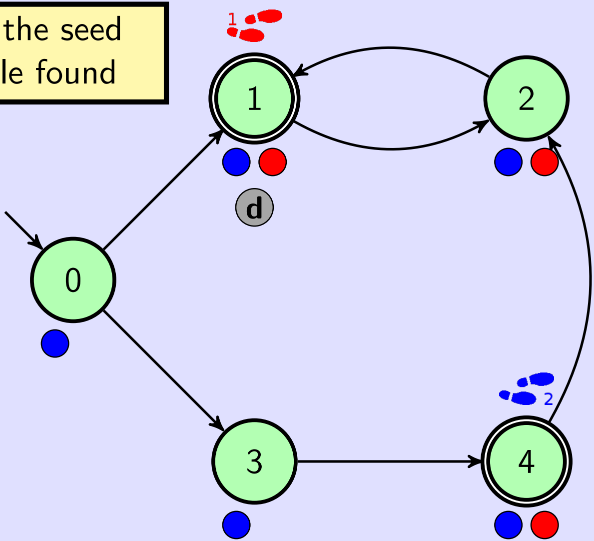


Example



Example

thread 1 reaches the seed
⇒ accepting cycle found



Time complexity

- in the worst case, a thread will explore each state of the graph
- then mc-ndfs is equivalent to spawn p unsynchronised instances of ndfs
- and we do not gain anything through multi-threading
- "good" input graphs: graphs clustered in many small SCCs (or acyclic)
  - with randomization threads visit different SCCs $\Rightarrow$ no/few state revisits
- "bad" input graphs: graphs with a single SCC

  $\Rightarrow$ lot of state revisits

Overview

Experimentation context

- prototype implementation in C on top of the pthread library
- algorithms exeperimented: mc-ndfs and map
- input models from the BEEM database:
  `http://anna.fi.muni.cz/models`
- 163 graphs with more than 10^6 states
  - 44 do not have an accepting cycle
  - 119 do have one
- out of the 119 “positive” graphs the accepting cycle was trivial to found
  - ndfs could report it after the visits of hundred states at most

  $\Rightarrow$ using a multi-core algorithm did not make sense
  - in 6 cases, an accepting cycle was hard to find and mc-ndfs could report it much faster

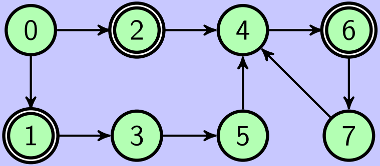
$\Rightarrow$ next we only report experiments on “negative” graphs

A Brief Overview of MAP

- MAP assumes a total order relation $>_S$ on states
- $map(s)$ is the Maximal Accepting Predecessor of s
  - the biggest accepting state (w.r.t. $>_S$) that is backward reachable from s
- $map(s)$ can be computed in $\mathcal{O}(a \cdot (n + m))$ using a modified BFS
- trivially: $map(s) = s \Rightarrow s$ is part of an accepting cycle
- but the converse is not true
- MAP will then relaunch the search after the deletion of some states
- algorithm stops when
  - an accepting cycle is found
  - or all accepting states have been deleted
- BFS is easy to distribute $\Rightarrow$ MAP is suited for parallel architectures

A Brief Overview of MAP — An Example

Our input graph



A Brief Overview of MAP — An Example

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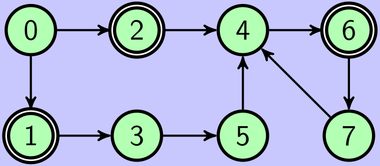
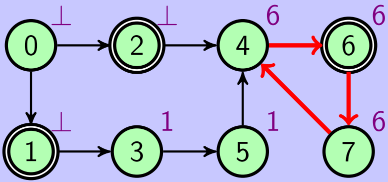


MAP with $6 >_S 2 >_S 1$



A Brief Overview of MAP — An Example

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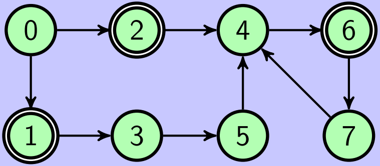

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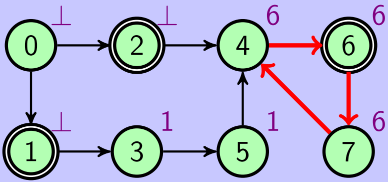
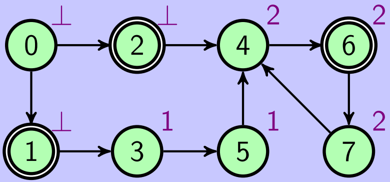
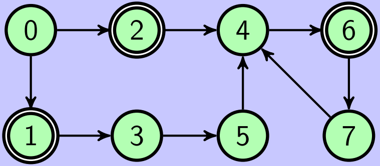


MAP with $2 >_S 1 >_S 6$

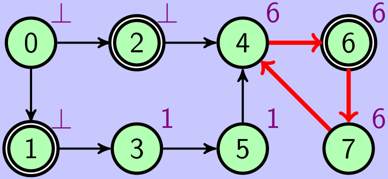


A Brief Overview of MAP — An Example

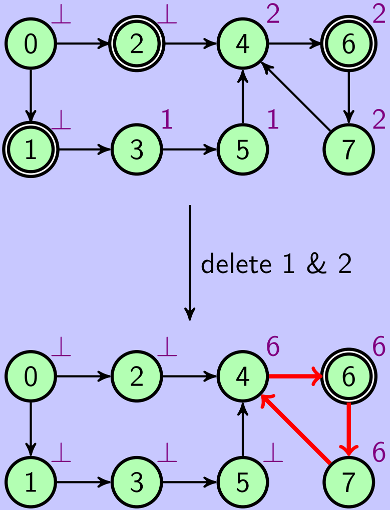
Our input graph



MAP with $6 >_S 2 >_S 1$



MAP with $2 >_S 1 >_S 6$



Evaluation Methodology

- we considered the following performance criterion

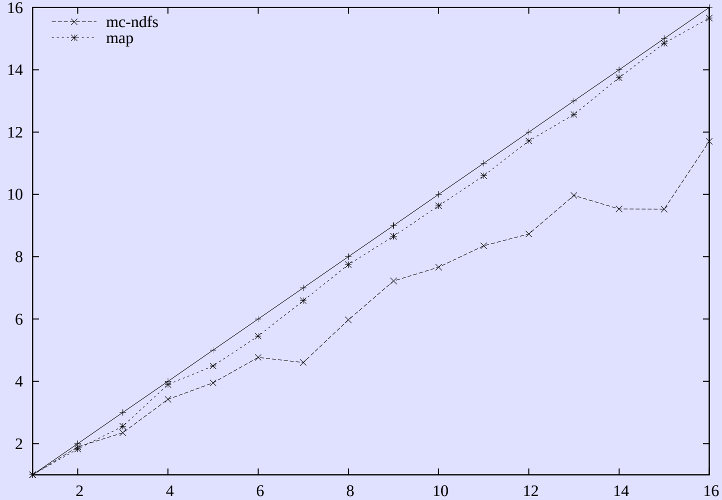
$$\max_{t \in \text{threads}} (\text{number of states explored by } t)$$

- for several reasons:
  - the graph explored was given explicitly (stored on disk)
    - all time consuming operations (computing successors, serialising states) were already done
    - synchronisations dominate the whole exploration time

  $\Rightarrow$ the time performance for both map and mc-ndfs were rather bad
  - it is implementation independent
  - it gives a better idea on the "theoretical" performance than time
- acceleration for N threads is measured as

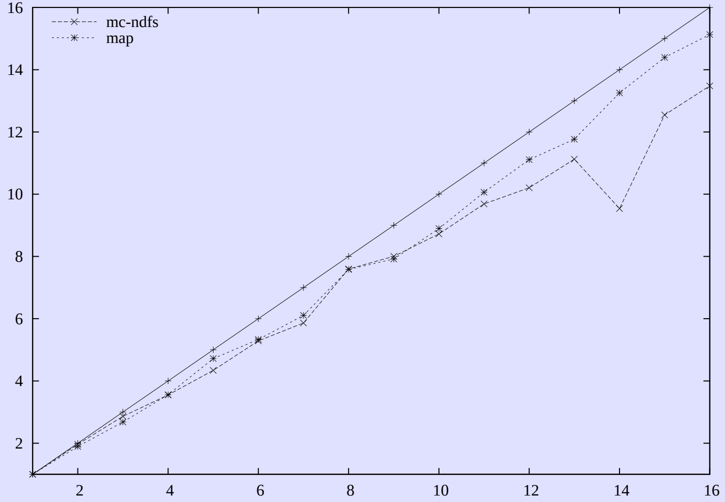
$$\frac{\text{performance for 1 thread}}{\text{performance for } N \text{ threads}}$$

Acceleration of MC-NDFS and MAP



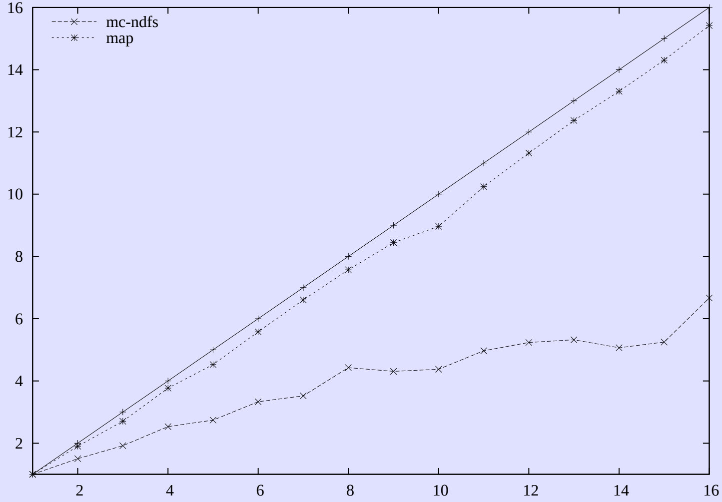
- model: **pgm_protocol** (pragmatic multicast protocol)
- property: every packet loss is followed by a negative ack
- graph size: 7,233,361 nodes

Acceleration of MC-NDFS and MAP



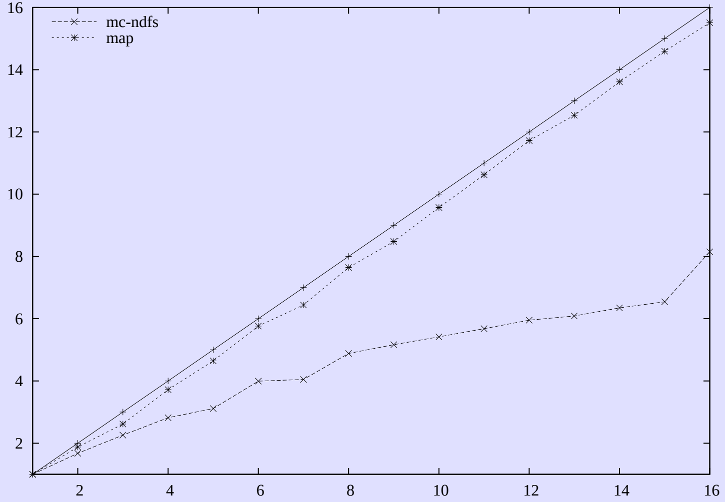
- model: **leader_filters** (election protocol)
- property: a leader is eventually elected
- graph size: 26,302,351 nodes

Acceleration of MC-NDFS and MAP



- model: **lup** (shared memory model)
- property: processor 0 will eventually have access to RAM
- graph size: 34,425,340 nodes

Acceleration of MC-NDFS and MAP



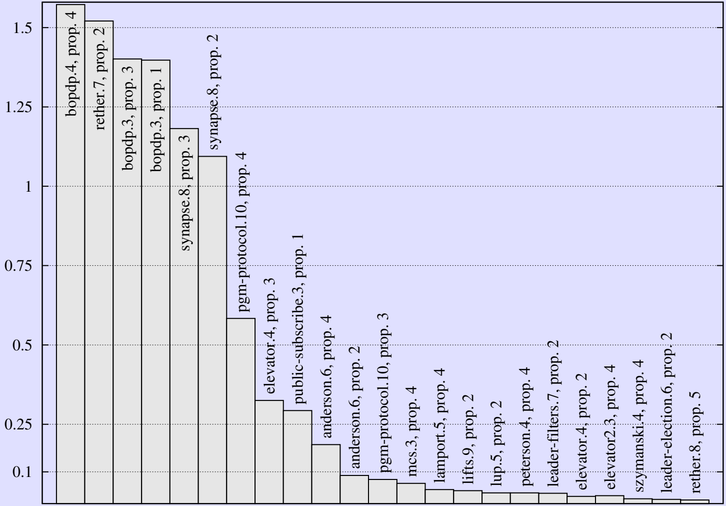
- model: **publish_subscribe** (Publish/subscribe notification protocol)
- property: ???
- graph size: 1,977,587 nodes

Acceleration of MC-NDFS and MAP

Conclusions

- mc-ndfs can clearly not compete with map on that point
- map: excellent accelerations in all situations
- mc-ndfs: the graph structure influences the acceleration
  - **pgm_protocol** and **leader_filters**: many small SCCs
    $\Rightarrow$ good acceleration
  - **lup** and **publish_subscribe**: one large SCC
    $\Rightarrow$ redundant revisits by different threads

Absolute Performances of MC-NDFS and MAP



- data plotted: $\frac{\text{performance of map}}{\text{performance of mc-ndfs}}$ for 16 working threads
- example: for graph **bopdp.4, prop. 4**, map is potentially 1.5 $\times$ faster

Absolute Performance of MC-NDFS and MAP

Conclusion

- we have seen that map clearly wins w.r.t. acceleration
- but it has a polynomial complexity in $a^2 \cdot (n + m)$
- so mc-ndfs is usually more efficient than map
- map is better than mc-ndfs when the graph has few/no accepting states

$\Rightarrow$ map is then equivalent to a parallel BFS

Overview

To sum up

- we have introduced mc-ndfs an LTL model checking algorithm for multi-core computers
- mc-ndfs is an adaptation of the sequential ndfs algorithm
- its principle
  - launch multiple threads executing a modified ndfs
  - each thread visits the graph in a random way
  - conflicts are not prevented but fixed a posteriori
  - (main) modification to ndfs: relaunch a red DFS in a safe mode when a conflict is detected
- perfomances largely depends on the graph structure
- but on some graphs we observed good accelerations

Perspectives

More experimentations

- with other kinds of models (e.g., Petri nets)
- comparison with other multi-core algorithms (e.g., bledge-otf)

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- we observed good "theoretical" accelerations with our prototype
- but not necessarily reflected in the time performance

Perspectives

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Better implementation

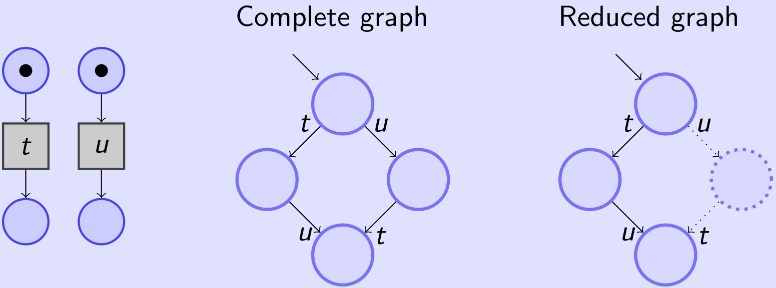
- we observed good "theoretical" accelerations with our prototype
- but not necessarily reflected in the time performance

Combination of mc-ndfs with other reduction techniques

- partial order reduction
- state caching

Partial order reduction

- the representation of interleaving is a major source of state explosion

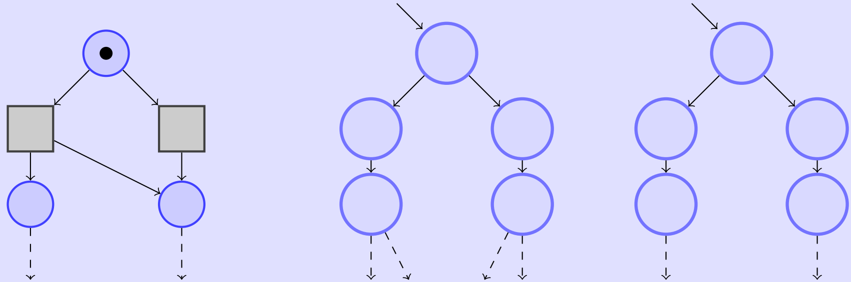


sufficient to execute $t.u$ if we are looking for deadlock states

- idea: perform a **selective** search to build a **reduced** graph
  1. perform a classical search, e.g., depth or breadth first
  2. at each state s, compute a **persistent** set of actions P
  3. only execute the actions of P and delay the execution of the others

The ignoring problem

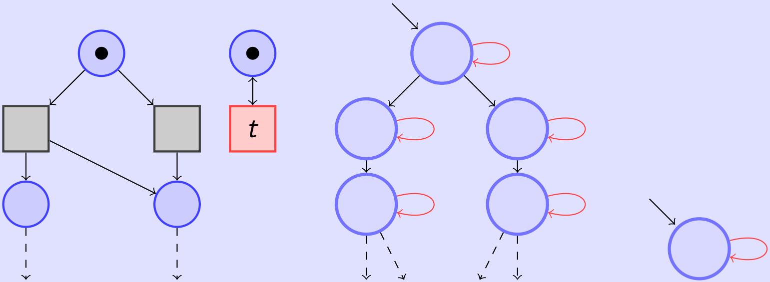
Net	Complete graph	Reduced graph



let's assume both graphs are equivalent with respect to a property Ψ

The ignoring problem

Net | Complete graph | Reduced graph



the reduced graph is useless (only useful if Ψ = deadlock freeness)
all transitions but t are infinitely delayed

The ignoring problem

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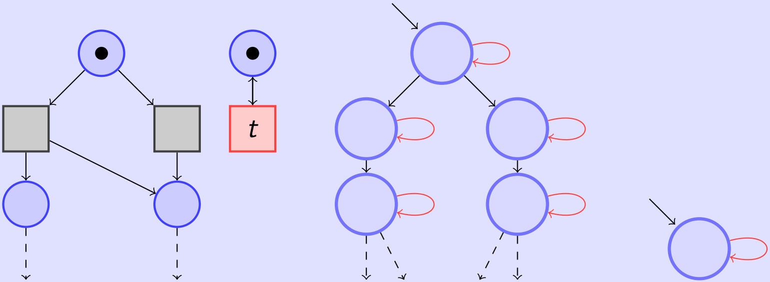

the reduced graph is useless (only useful if Ψ = deadlock freeness)
all transitions but t are infinitely delayed

we have to ensure that mc-ndfs does not ignore transitions