Concurrent Algorithms and Data Structures for Model Checking

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Smart Parallel Algorithms for Model Checking

Model checking requires the exploration of very large, implicit graphs
These graphs are generated from specifications (models, programs)

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Challenge: efficiency = time-optimal + linear speedup

Opportunities and obstacles in parallel model checking

Distributed Model Checking

- More memory is available (NoW = Network of Workstations)
- Price: communication costs
- Main limitation: latency and throughput of the network
- Redesign algorithms (load balancing, latency hiding, speculation)

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Multi-core Model Checking

- State space is available in shared memory: efficient communication
- Main limitation: memory bus contention, cache coherence, locking
- Graphs: irregular memory access (hash tables, BDDs)
- Computer architecture: from SMP to NUMA
- Efficiency: lock-free (CAS, memory barriers), be cache-line aware

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In both cases, thorough experimental evaluation is important

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- 2011 Laarman & vdPol, Multi-core Nested DFS
- 2013 Van Dijk & vdPol, Scalable multi-core BDD algorithms
- 2016 Bloemen & vdPol, Multi-core DFS SCC algorithm

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Alfons Laarman: Parallel Nested Depth-First Search (2010-2014)

- lock-free hashtable, state compression (make-over: Freark vd Berg)
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- parallel symbolic reachability, bisimulation minimisation, saturation
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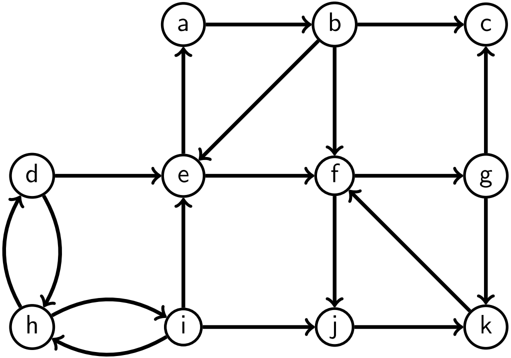
Vincent Bloemen: Parallel Strongly Connected Components (2016-2019)

- based on DFS and sharing info on partial SCCs
- concurrent Union-Find structure + iterable cyclic list
- LTL model checking with Büchi automata, Rabin automata, etc.

Overview

Strongly Connected Component (SCC)

Setting: finite graph with directed edges
SCCs: maximal components of $\twoheadrightarrow \cap \twoheadleftarrow$



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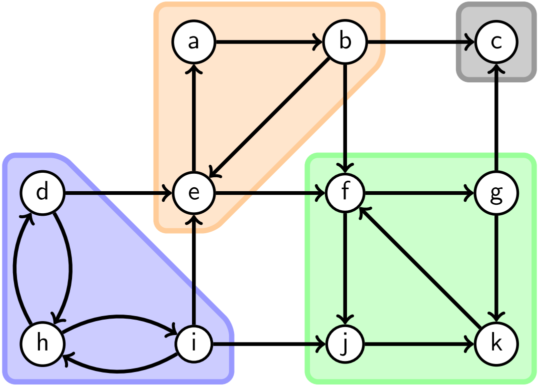

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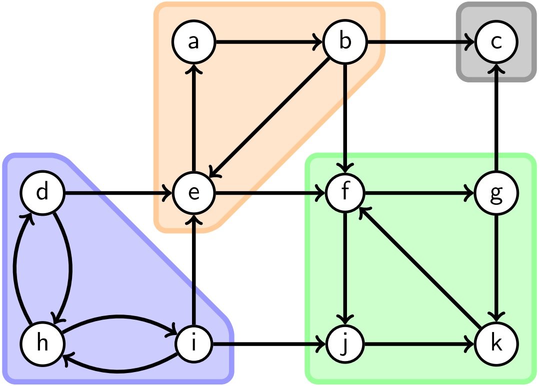
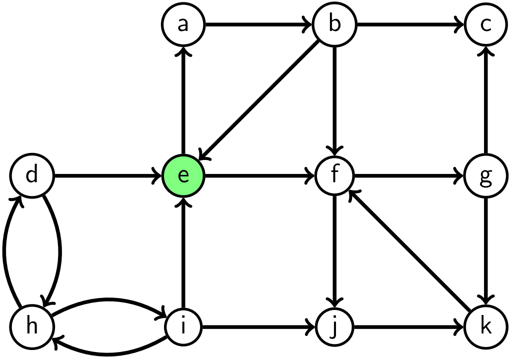


Applications: LTL model checking, fairness, evaluation of Markov Chains

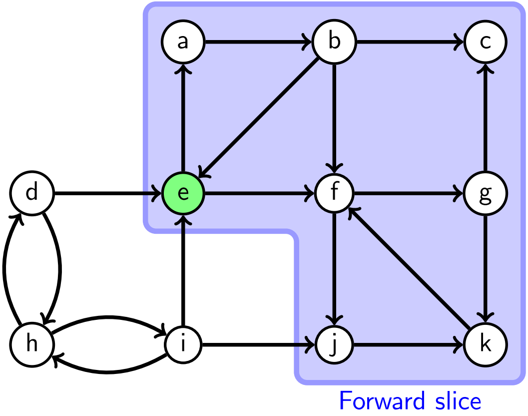
Forward-Backward (FB) parallel SCC Algorithm

1. Select a pivot node



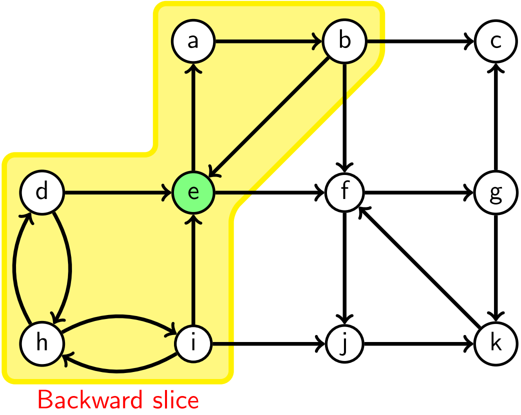
Forward-Backward (FB) parallel SCC Algorithm

2. Compute its forward reachable set (F)

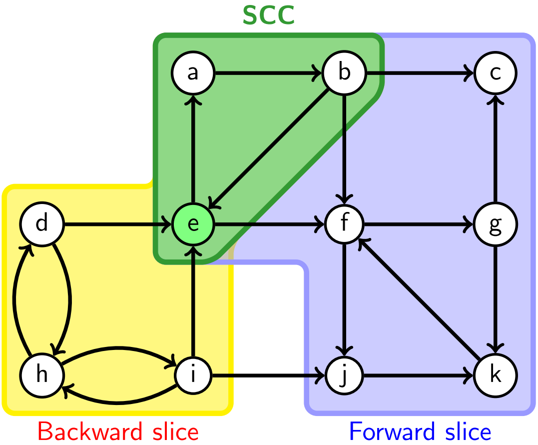


Forward-Backward (FB) parallel SCC Algorithm

3. Compute its backward reachable set (B)



4. The intersection F ∩ B is the SCC of the pivot



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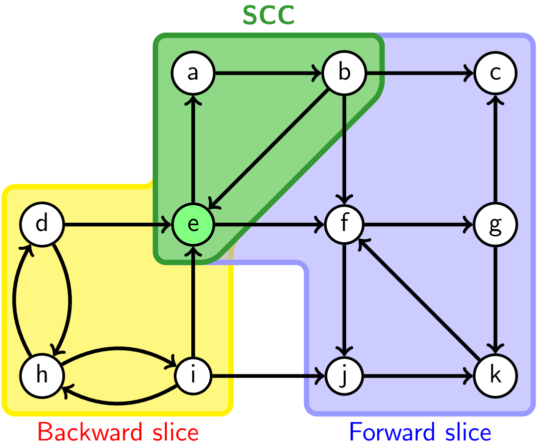
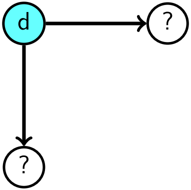


Remaining slices can be processed independently in parallel

Finding SCCs on-the-fly (path-based algorithm)

For model checking, an on-the-fly SCC algorithm is preferable:

- bug finding: early termination when a bug in the model is detected
- portability: we restrict model access to a next-state function



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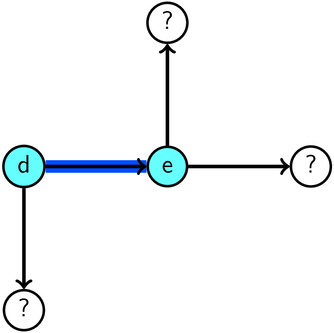

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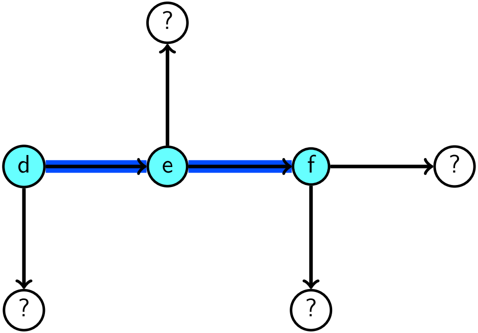

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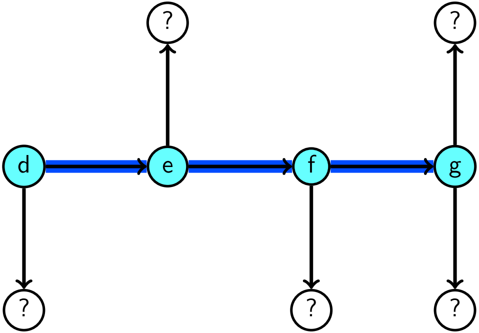

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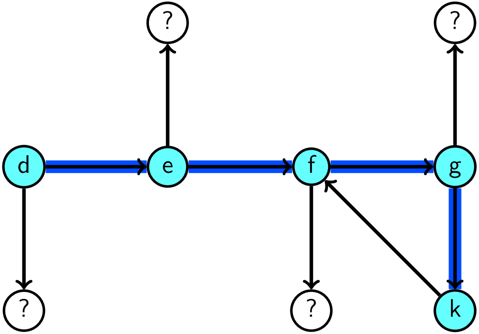

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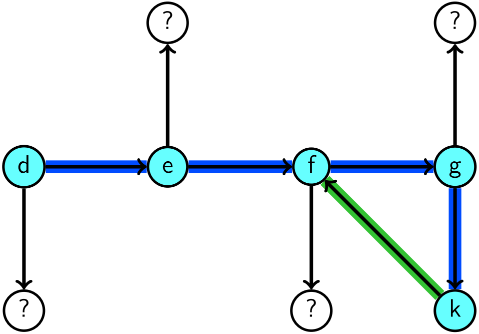

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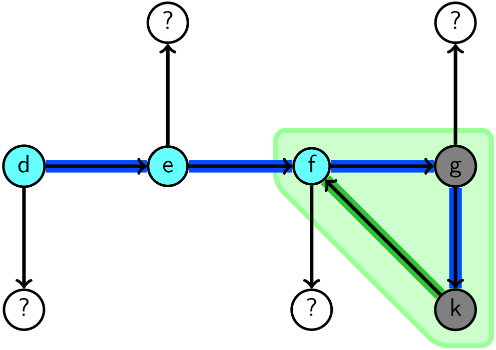
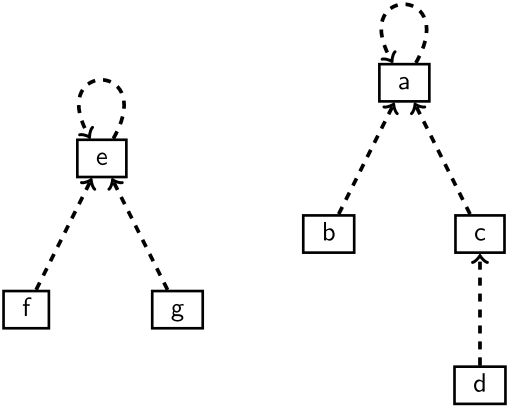


NB: this is not yet necessarily a maximal SCC,
since its successors are not completely explored

Maintain (partial) SCCs in a Union-Find data structure

Union-Find structure *[Tarjan, van Leeuwen, JACM 1984]*:

- supports disjoint subsets, which can be merged
- basic functions: Union and Find (unique representative)



Reversed forest, nodes direct towards their representative root

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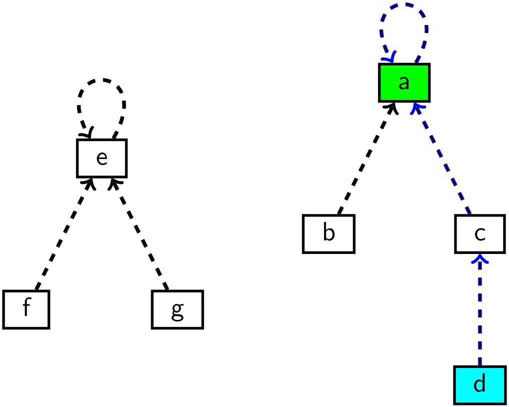

`Find(d)`: recursively searches the parent edges to find the root

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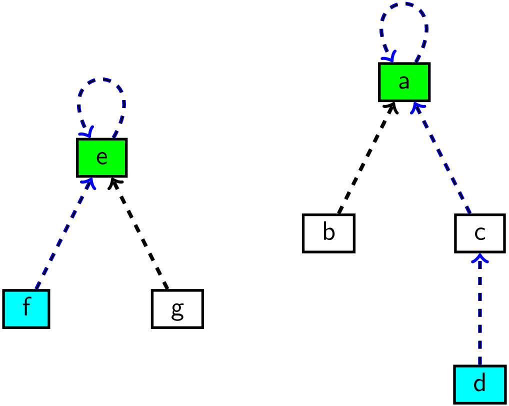

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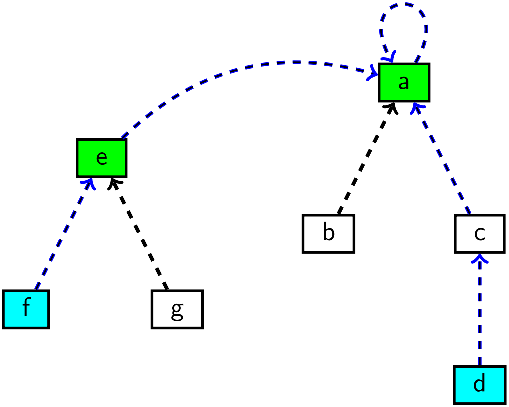

`Unite(f,d)`: Find the roots of `f` and `d`, and update one of them

Dijkstra's SCC Algorithm [1976]

Uses *stack* R (push, pop, top) and *disjoint-set* S (union, find, enum)
Also maintains sets *Visited* and *Explored*, initially $\varnothing$

$\dots$
$\dots$
w
$\dots$
$\dots$
v

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procedure SCC(v)
  Visited := Visited ∪ {v}
  R.push(v)
  for each w ∈ next_state(v)

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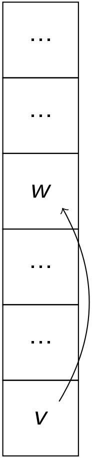

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    else                            // cycle found

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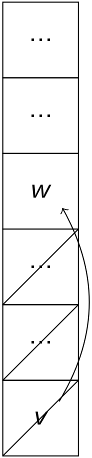

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        S.union(R.pop(), R.top())
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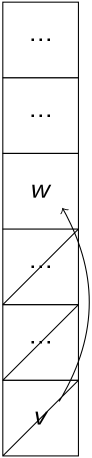

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· · · · S.union(R.pop(), R.top())
· if v = R.top() then · · · · ·  // completed SCC
· · report SCC S.enum(v)
· · Explored := Explored ∪ S.enum(v)
· · R.pop()
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Comparing SCC algorithms

Tarjan's SCC algorithm [1972]

- Worst case $O(m + n)$
- On-the-fly
- Inherently DFS

Forward-Backward algorithm

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- Requires predecessors
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Variants (DFS-based):

- double DFS (transposed graph)
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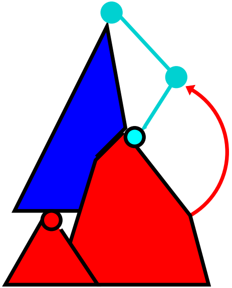
Complexity theory of parallel graph algorithms:

- Reif (1985): *Depth-First Search is inherently sequential* (P-complete)
- Amato (1993): SSSP in $O(log^2(n))$ time on $O(n^{2.376})$ processors

Intermezzo: Parallel Random Nested-DFS (ATVA 2011, 2012)

Inspiration: Swarmed Verification (Holzmann, Spin) for bug finding



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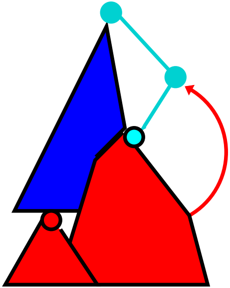

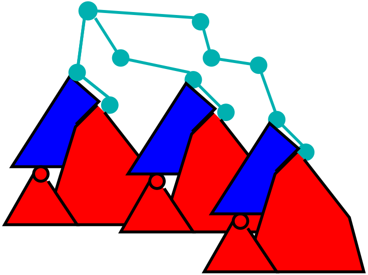

Nested Depth-First Search for LTL model checking

- Every worker performs its own NDFS in a randomized direction
- Parallel search speeds up finding bugs only: duplicate work

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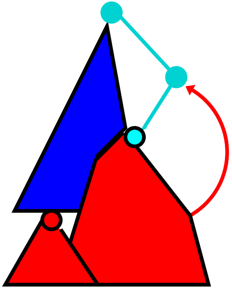
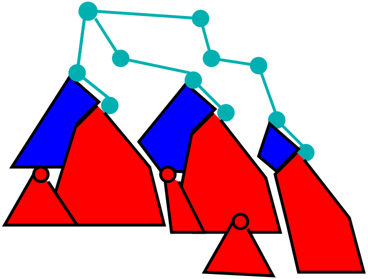


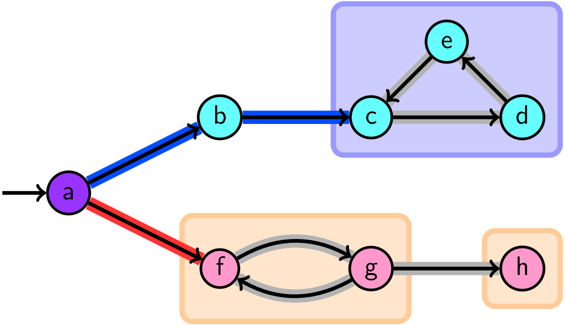


Nested Depth-First Search for LTL model checking

- Every worker performs its own NDFS in a randomized direction
- Parallel search speeds up finding bugs only: duplicate work
- EP 2011: Share much, repair violations of DFS order: sequential work
- LvdP 2011: Share less, avoid violations of DFS order: some locking

Parallel Random DFS for SCCs

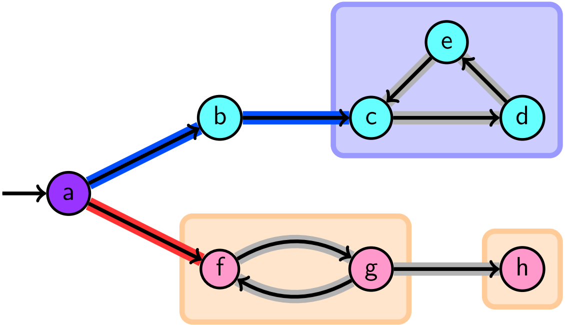
Parallel DFS + random successor order + sharing information on SCCs



What happens if two workers start working on the same SCC?

Parallel Random DFS for SCCs

Parallel DFS + random successor order + sharing information on SCCs

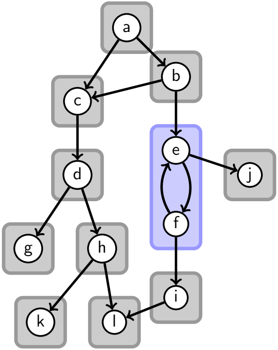


What happens if two workers start working on the same SCC?

- G. Lowe (TACAS'14): suspend and sequential repair procedure
- E. Renault et al. (TACAS'15): share complete SCCs only
- V. Bloemen et al. (PPoPP'16): share partial SCCs as well

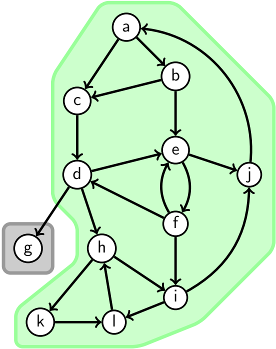
Handling Small and Large SCCs Sequentially

Small SCCs



Parallelizes well

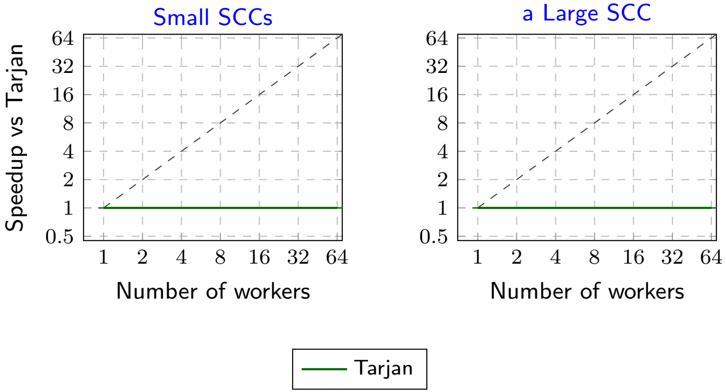
Large SCCs



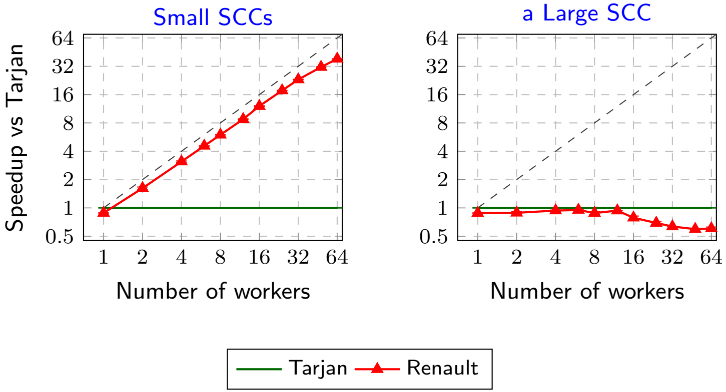
No performance gain

Bottom line: we cannot afford to handle single SCCs sequentially

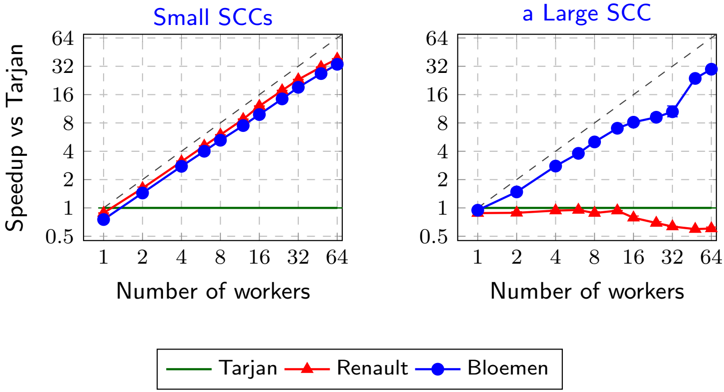
Speedup in practice



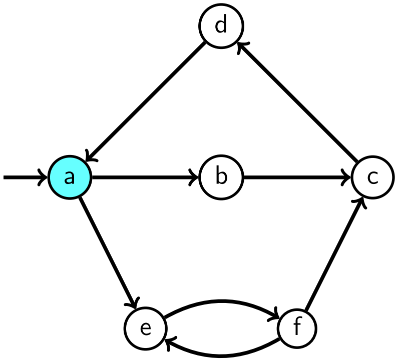
Speedup in practice



Speedup in practice



UF-SCC: Communicate partially found SCCs [Bloemen]



Blue worker happens to visit $a \to b \to c \to d$

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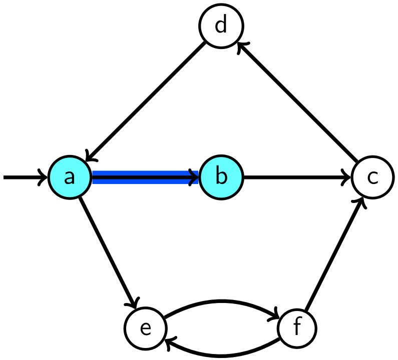

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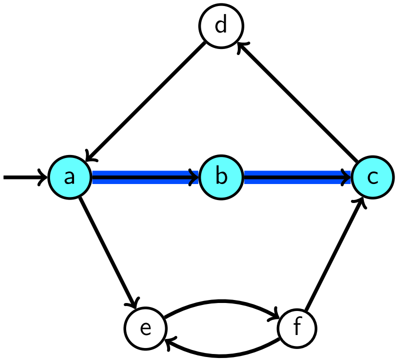

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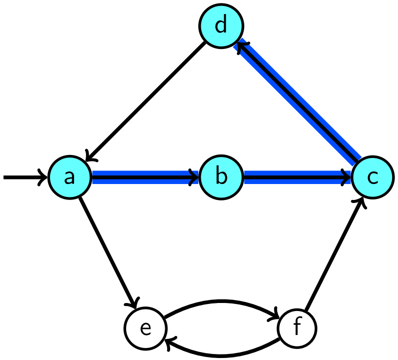

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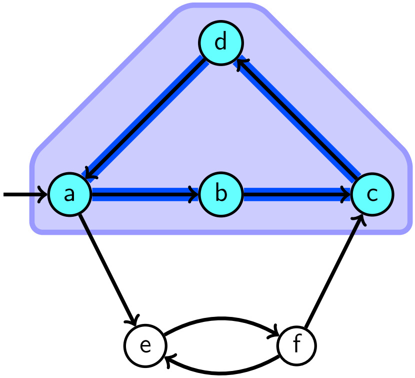
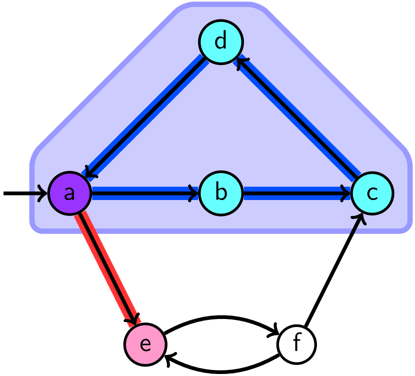


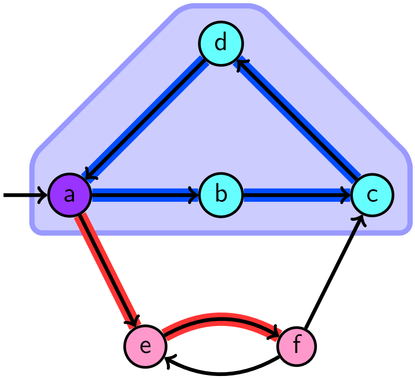
Blue worker detects and unites partial SCC $\{a, b, c, d\}$

UF-SCC: Communicate partially found SCCs [Bloemen]



Red worker happens to visit $a \rightarrow e \rightarrow f$

UF-SCC: Communicate partially found SCCs [Bloemen]



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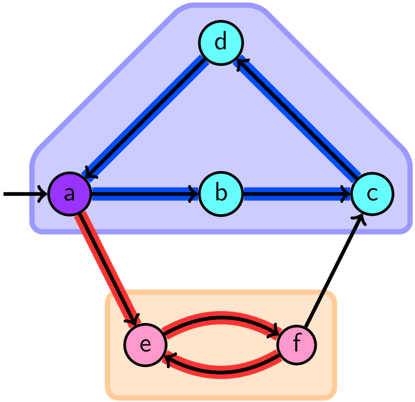

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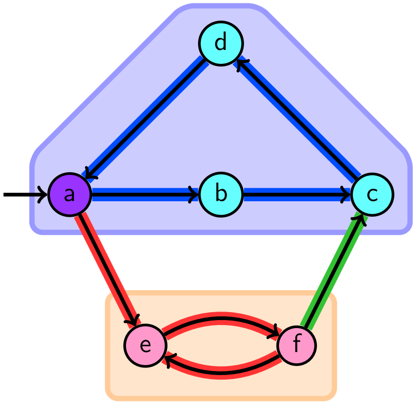
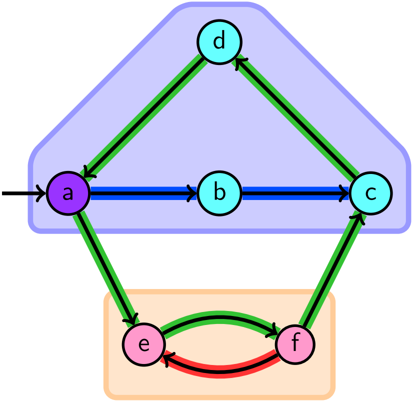


Red worker continues exploration $f \to c$

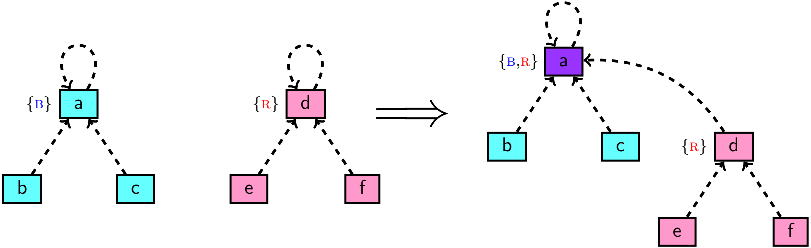
UF-SCC: Communicate partially found SCCs [Bloemen]



But how does Red worker know that it visited a state “equivalent” to c?

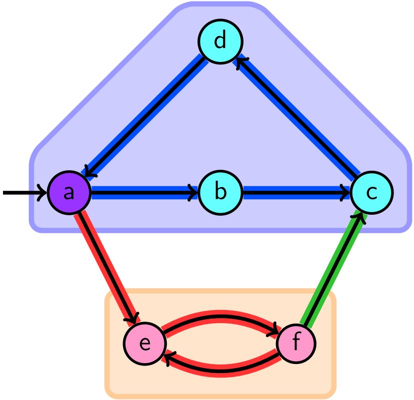
Union-Find with a worker set

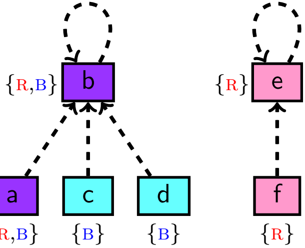
Store a bit-set of worker IDs in the union-find roots



Union-Find with a worker set

Check if the partial SCC of the successor has been visited before





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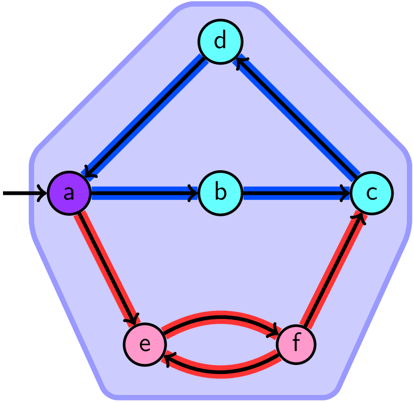

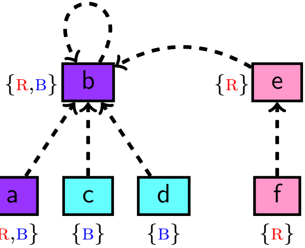

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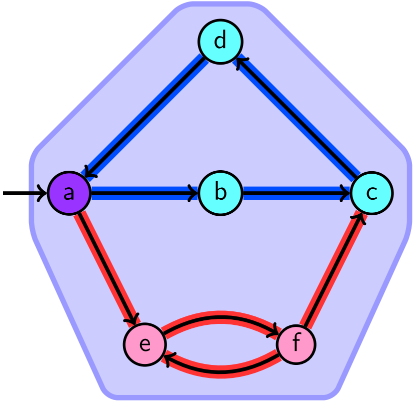
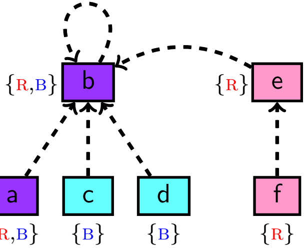
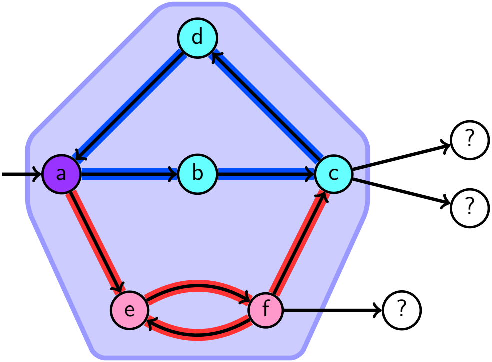




But how do we know when the SCC is complete?

Distinguish fully explored states

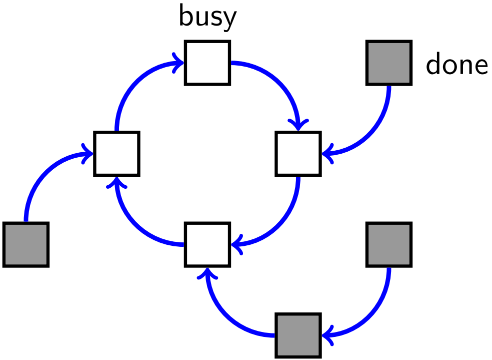
- Track which states of the SCC still have to be explored
  - An SCC is complete if all its states have been fully explored
- Evenly distribute the remaining work
  - Otherwise one worker may end up doing all the work

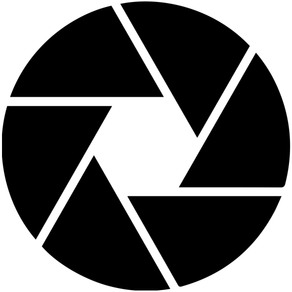


Cyclic list of BUSY states

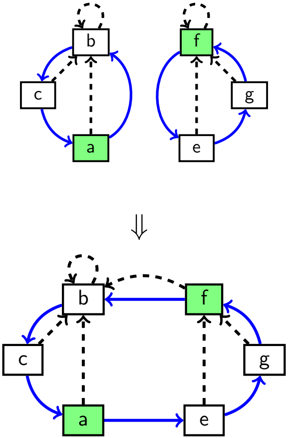
Keep track of the TODO list of BUSY states

- BUSY: There may be some unexplored successors from this state
- DONE: This state has been fully explored by some worker
- Workers can concurrently pick states from the cyclic list





List operations

Merge lists (Union)	Remove element


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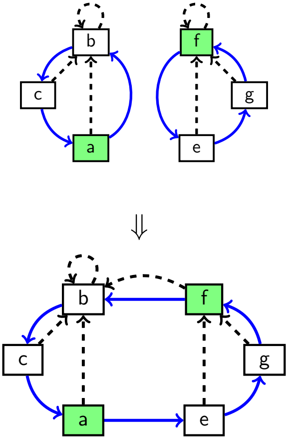
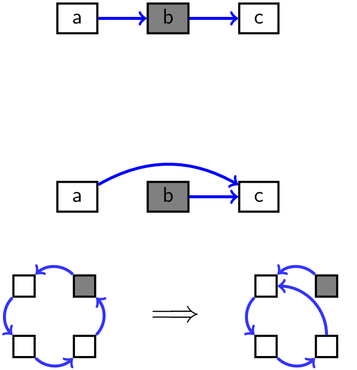


Remove element



Algorithm: code for worker p

Uses local *stacks* R_p (push, pop, top)
and shared *disjoint-set* S (union, find, claim, equal, cyclic list)

```
procedure UFSCC_p(v)
· S.claim(v,p)     // Add p to workers, v to cyclic list
· R_p.push(v)
· while v' := S.PickFromList(v)
· · for each w ∈ randomize(next_state(v'))
· · ·
· · ·
· · ·
· · ·
· · ·
· ·
· ·
· · S.RemoveFromList(v')
· if v = R_p.top() then report R_p.pop()  // report the SCC
```

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· S.claim(v,p)       // Add p to workers, v to cyclic list
· R_p.push(v)
· while v' := S.PickFromList(v)
· · for each w ∈ randomize(next_state(v'))
· · · if w ∈ DEAD                         // ignore completed SCC
· · · · then continue
· · · else if p ∉ S.find(w)              // state yet unseen by p
· · · · then UFSCC_p(w)
· · · else
· · · · while ¬S.equal(v,w)              // merge states on cycle
· · · · · S.union(R_p.pop(), R_p.top())
· · S.RemoveFromList(v')
· if v = R_p.top() then report R_p.pop()  // report the SCC
```

Time Complexity and Speed-Up

- n: number of states (nodes), m: number of transitions (edges)
- $\alpha(n)$: inverse of Ackermann function (amortized complexity of UF)
- p: number of workers

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In the worst case, all workers visit the whole graph in lockstep, so total amount of work is $O((m+n).\alpha(n).p)$: linear-time, but no speed-up

Model checking graphs are "broad", so workers spread out evenly.
Observed wall clock: $O((m+n).\alpha(n)/p)$: linear-time and linear speed-up

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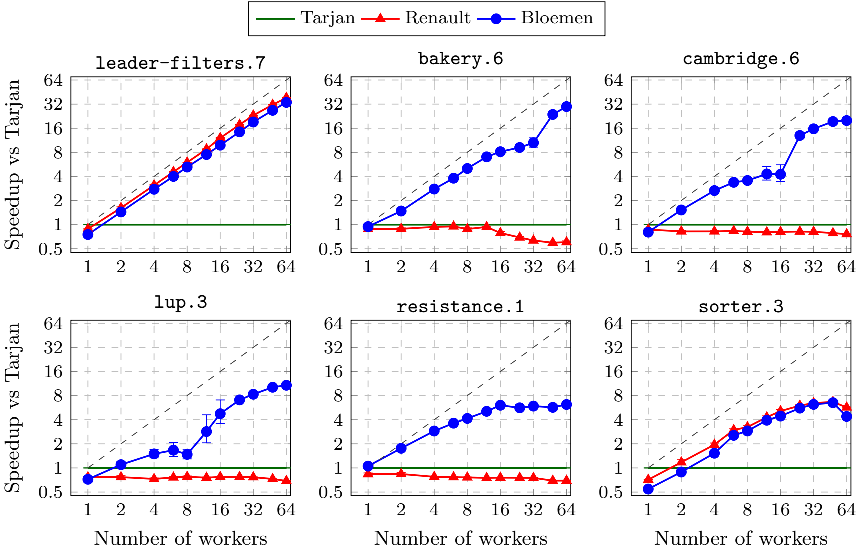
Can we guarantee even more? Maybe!

- S.V. Jayanti, R.E. Tarjan, E. Boix-Adserà [PODC'19]
  Randomized Concurrent Set Union and Generalized Wake-Up

reports the first concurrent union-find algorithm with a total work complexity that grows sublinear in p, the number of processes.

Speedup graphs of selected BEEM models



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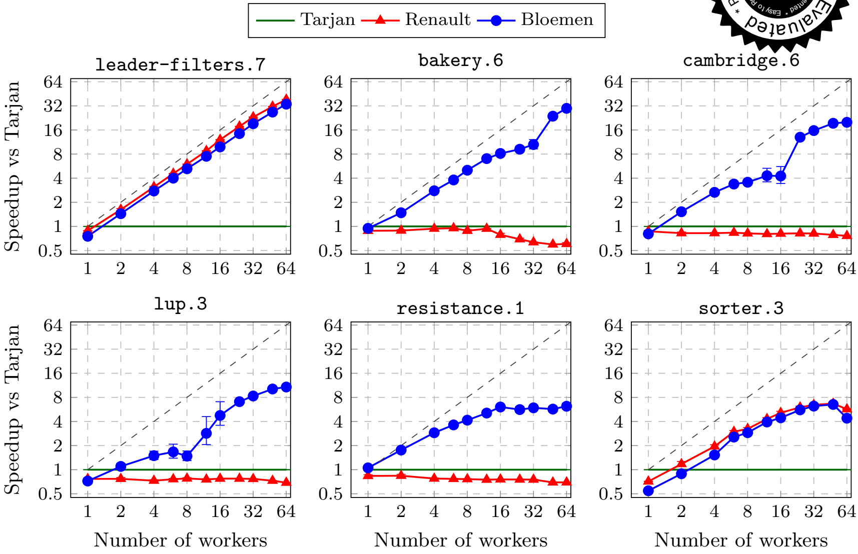
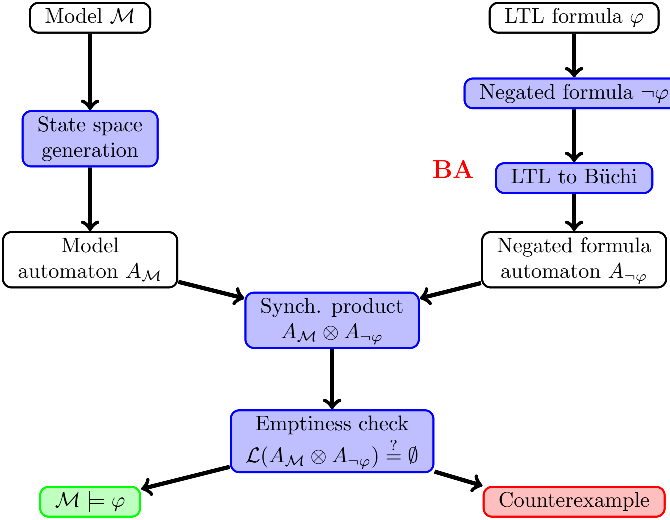


Overview

1. Introduction
2. Strongly Connected Components
   - A simple parallel SCC algorithm
   - Dijkstra's sequential SCC algorithm
   - A parallel DFS algorithm for SCCs
3. Multicore Model Checking
   - Explicit-state LTL model checking
   - Symbolic model checking
   - LTSmin: high-performance model checker
4. Conclusion

Recall: Automata-based LTL model checking



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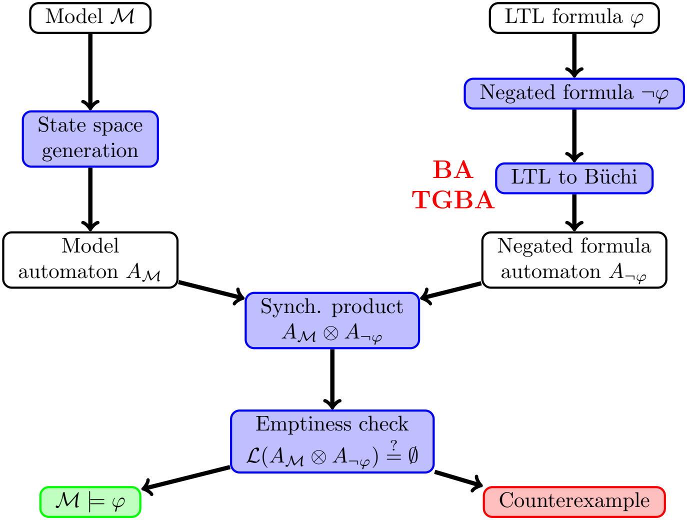

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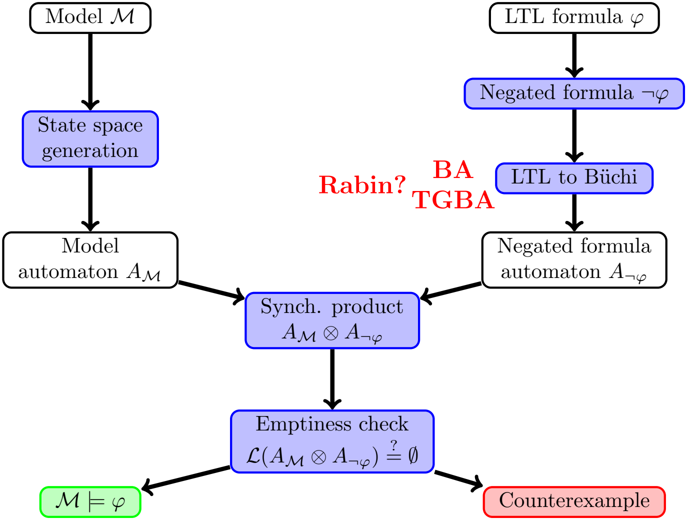
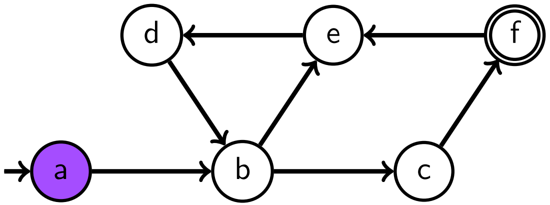


LTL model checking reduces to the following graph problem:
Find a reachable accepting SCC in a Büchi-automaton



Blue worker explores $a \rightarrow b \rightarrow e \rightarrow d$

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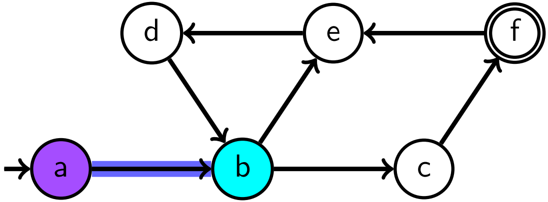

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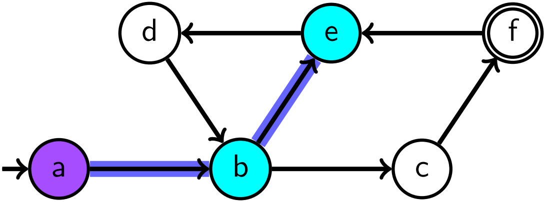

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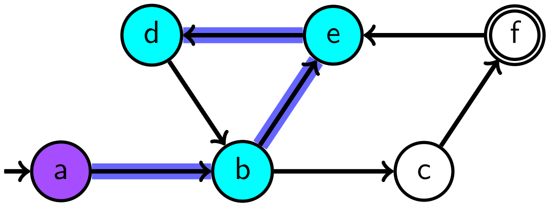

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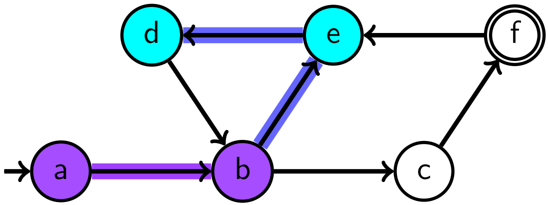

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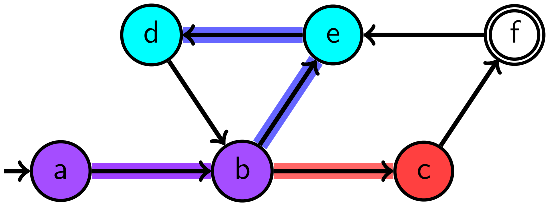

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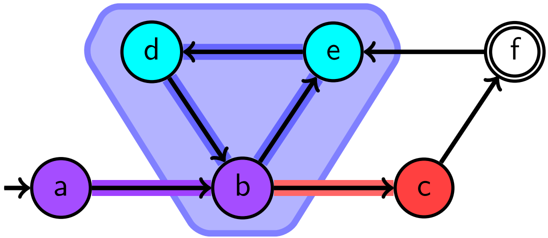

Blue worker detects and shares partial SCC $\{b, d, e\}$

UFSCC for LTL Model Checking with Büchi Automata

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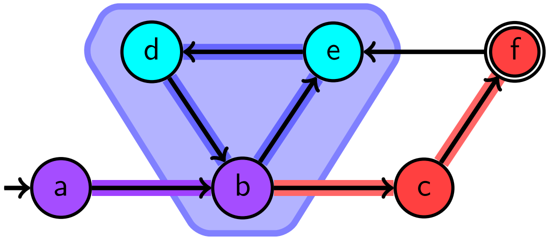

Red worker detects complete, accepting SCC $\{b, c, d, e, f\}$

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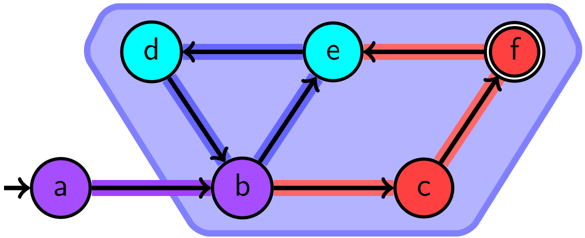

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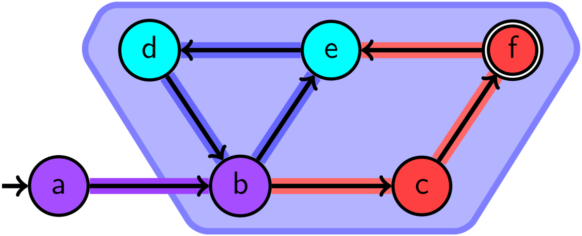
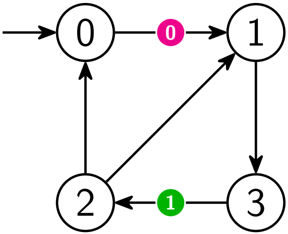


Accepting cycle has been found, while no single worker traversed it!

Accepting Cycle for TGBA

Transition-based Generalized Büchi Automata

$\mathrm{Inf}(⓿) \wedge \mathrm{Inf}(❶)$



Advantage: TGBA are more concise and natural for LTL

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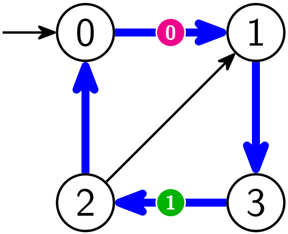

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Accepting Cycle for TGBA

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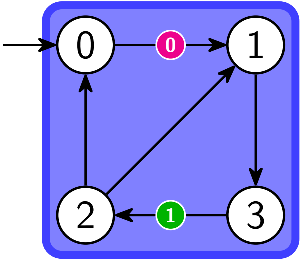
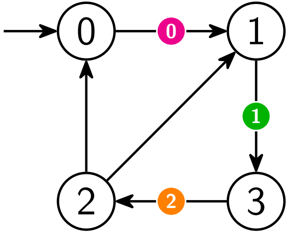


Advantage: TGBA are more concise and natural for LTL

Store all encountered accepting marks at the UF-root

Accepting Cycle for Generalized Rabin Automata

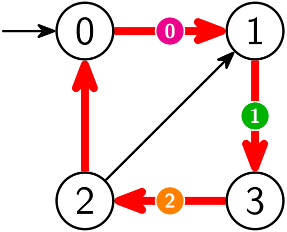
$$\mathsf{Fin}(⓿) \wedge \mathsf{Inf}(➊) \wedge \mathsf{Inf}(➋)$$



[Bloemen, Duret-Lutz, vdPol, SPIN 2017]
Can handle all Rabin conditions sequentially or in parallel

Accepting Cycle for Generalized Rabin Automata

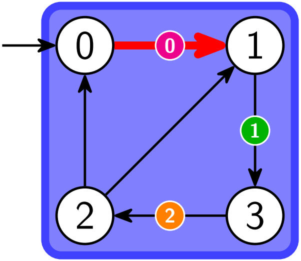
$$\mathrm{Fin}(⓿) \wedge \mathrm{Inf}(❶) \wedge \mathrm{Inf}(❷)$$



[Bloemen, Duret-Lutz, vdPol, SPIN 2017]
Can handle all Rabin conditions sequentially or in parallel

Accepting Cycle for Generalized Rabin Automata

$$\mathsf{Fin}(0) \land \mathsf{Inf}(1) \land \mathsf{Inf}(2)$$



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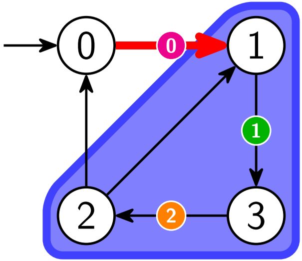
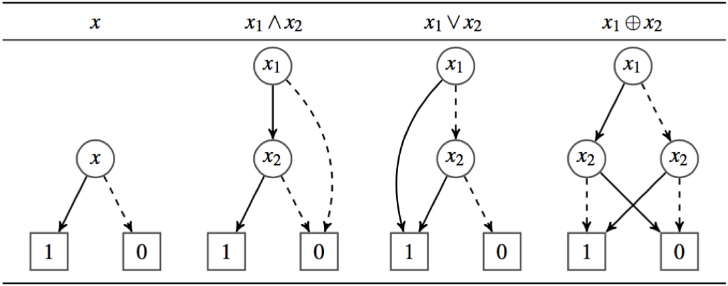


[Bloemen, Duret-Lutz, vdPol, SPIN 2017]
Can handle all Rabin conditions sequentially or in parallel
Adapt the UF-SCC procedure by postponing “fin”–labels

Binary Decision Diagrams

- Concise, canonical, representation for Boolean functions
- Used in Symbolic Model Checking to represent sets of states



Apply Operator on Binary Decision Diagrams

Towards Multi-Core BDD [Tom van Dijk]

Apply($\otimes, leaf_1, leaf_2$) $= leaf_1 \otimes leaf_2$

Apply($\otimes, B_1, B_2$) $=$

let $z = min(topvar(B_1), topvar(B_2))$

$L =$ Apply($\otimes, B_1|_{z=0}, B_2|_{z=0}$)

$H =$ Apply($\otimes, B_1|_{z=1}, B_2|_{z=1}$)

$R =$ MakeUniqueNode(z, L, H)

in R

- Two recursive calls

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$R =$ MakeUniqueNode(z, L, H)

store $(\otimes, B_1, B_2) \rightarrow R$ in cache

in R

- Two recursive calls
- MakeUniqueNode uses concurrent shared hashtable
- Caching uses concurrent lossy hashtable

Apply Operator on Binary Decision Diagrams

Towards Multi-Core BDD [Tom van Dijk]

Apply($\otimes, leaf_1, leaf_2$) $= leaf_1 \otimes leaf_2$
Apply($\otimes, B_1, B_2$) = if $(\otimes, B_1, B_2) \to R$ in cache, return R
Apply($\otimes, B_1, B_2$) =
    let $z = min(topvar(B_1), topvar(B_2))$
        L = spawn Apply($\otimes, B_1|_{z=0}, B_2|_{z=0}$)
        H = spawn Apply($\otimes, B_1|_{z=1}, B_2|_{z=1}$)
        R = MakeUniqueNode(z,sync L, sync H)
        store $(\otimes, B_1, B_2) \to R$ in cache
    in R

- Two recursive calls
- MakeUniqueNode uses concurrent shared hashtable
- Caching uses concurrent lossy hashtable
- Spawn/Sync requires a fine-grained task scheduler (deque)

Sylvan Framework for Multi-core Decision Diagrams

Features of Sylvan [https://github.com/utwente-fmt/sylvan]

- Support: BDD, Multiway/Multiterminal DDs, ZDDs, ...
- Programmable interface (C, C++, Python)
- Ported to RDMA: Multicore/Distributed [Wytse Oortwijn, SPIN17]

Missing: dynamic variable reordering

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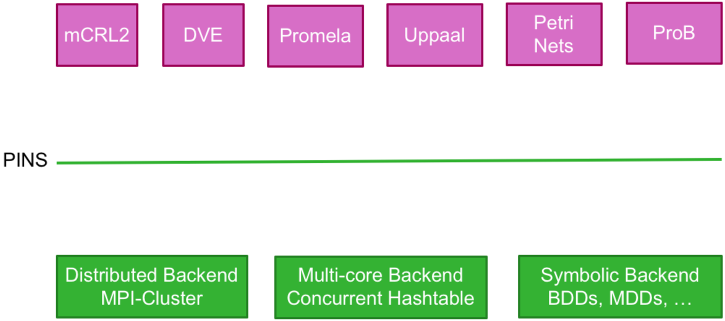
Applications

- Symbolic Reachability with BFS strategy and Saturation
- Symbolic Bisimulation Reduction / CTMC lumping
- Symbolic Parity Game Solving (Zielonka's algorithm)

LTSmin: high-performance model checker

LTSmin and its language-independent interface PINS

https://github.com/utwente-fmt/ltsmin



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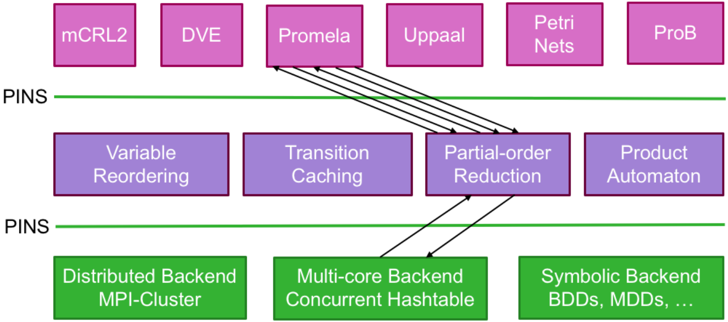

- Parallel LTL-X model checking with partial-order reduction
- Symbolic reachability with saturation and bisimulation reduction
- Distributed reachability and bisimulation reduction

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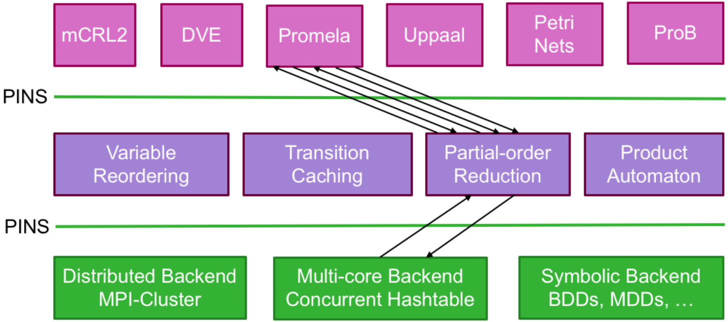

- Parallel LTL-X model checking with partial-order reduction
- Symbolic reachability with saturation and bisimulation reduction
- Distributed reachability and bisimulation reduction
- Competition Awards: RERS 2012, 2013, 2016; MCC 2016 gold in LTL

Conclusion

Concurrent Datastructures

- hash-tables, lossy cache, union-find, deque
- mostly lock-less, use CAS, NUMA-aware programming

Conclusion

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Parallel Algorithms, in particular parallel DFS-based

- Total amount of work: try to avoid duplicate work
- Speedup bottlenecks: try to avoid sequential repair
- Careful reconsider necessary invariants

Conclusion

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Parallel Algorithms, in particular parallel DFS-based

- Total amount of work: try to avoid duplicate work
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- Careful reconsider necessary invariants

Recent directions of interest

- GPU algorithms and implementations
- Parallel SAT/QBF solving
- Parallel parameter synthesis (probability, time)
- Parallel strategy synthesis for games

Literature: Overview and LTSmin tool

- J. Barnat, V. Bloemen, A. Duret-Lutz, Laarman, Petrucci, vd Pol, Renault
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- T. van Dijk, J. van de Pol
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- S. Blom, J. van de Pol, M. Weber [CAV'10]
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- A. Laarman, J. van de Pol, M. Weber [NFM'11]
  Multi-Core LTSmin: Marrying Modularity and Scalability
- G. Kant, A. Laarman, J. Meijer, J. vd Pol, S. Blom, T. v Dijk [TACAS'15]
  LTSmin: High-Performance Language-Independent Model Checking

Literature on parallel DFS-based SCC detection

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  Depth-First Search and Linear Graph Algorithms
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  Concurrent Depth-First Search Algorithms
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  Multi-Core On-The-Fly SCC Decomposition
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  Randomized Concurrent Set Union and Generalized Wake-Up.

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- O. Grumberg, T. Heyman, A. Schuster [CAV'01]
  Distributed Symbolic Model Checking for μ-Calculus
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- Tom van Dijk, Jeroen Meijer, Jaco van de Pol [TACAS'19]
  Multi-core On-The-Fly Saturation

Parallel SCC with UF and Cyclic List